Gravitational waves from the early Universe

Part 2

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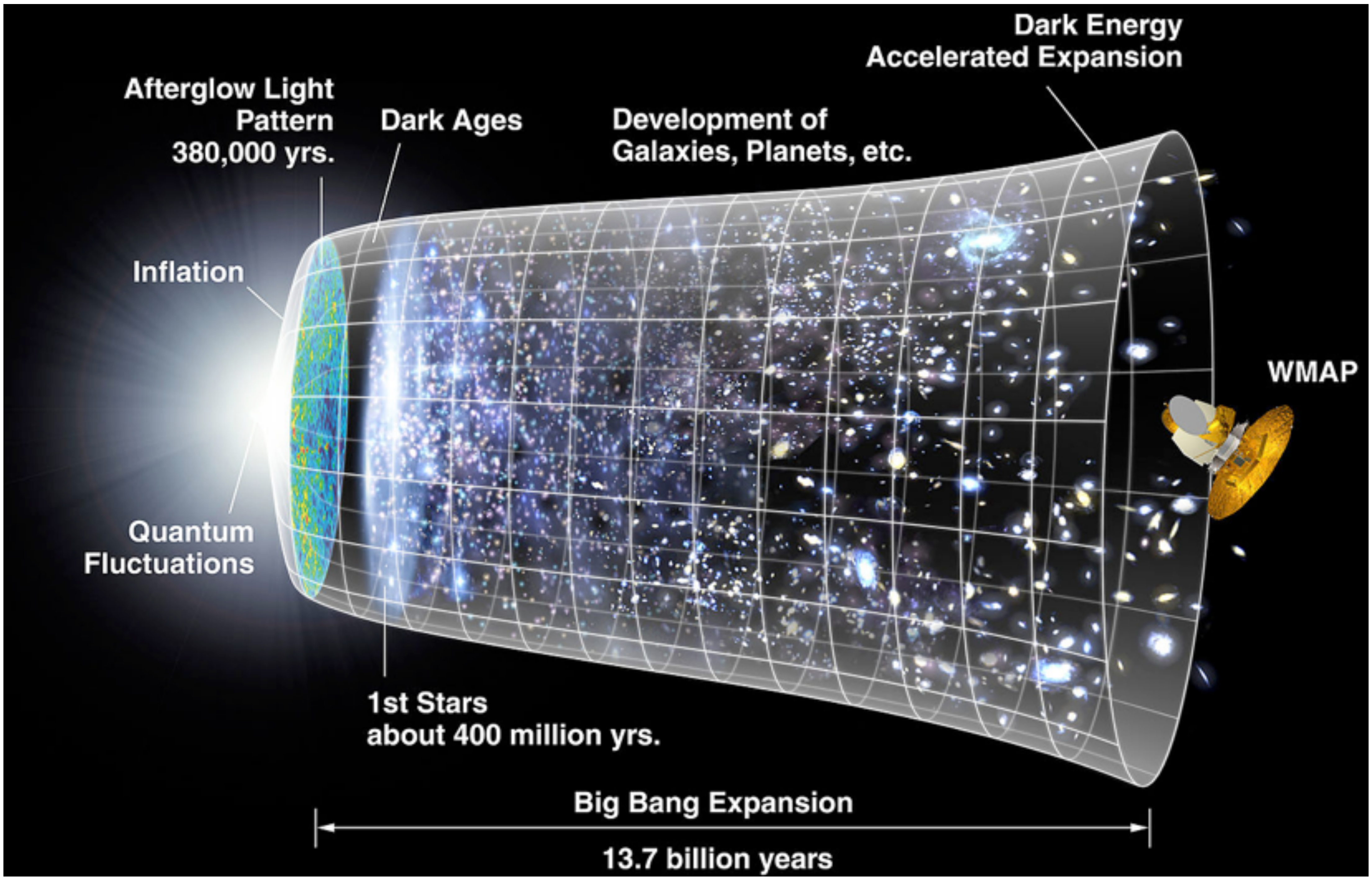
15 July 2017

GW mini-school@NTNU

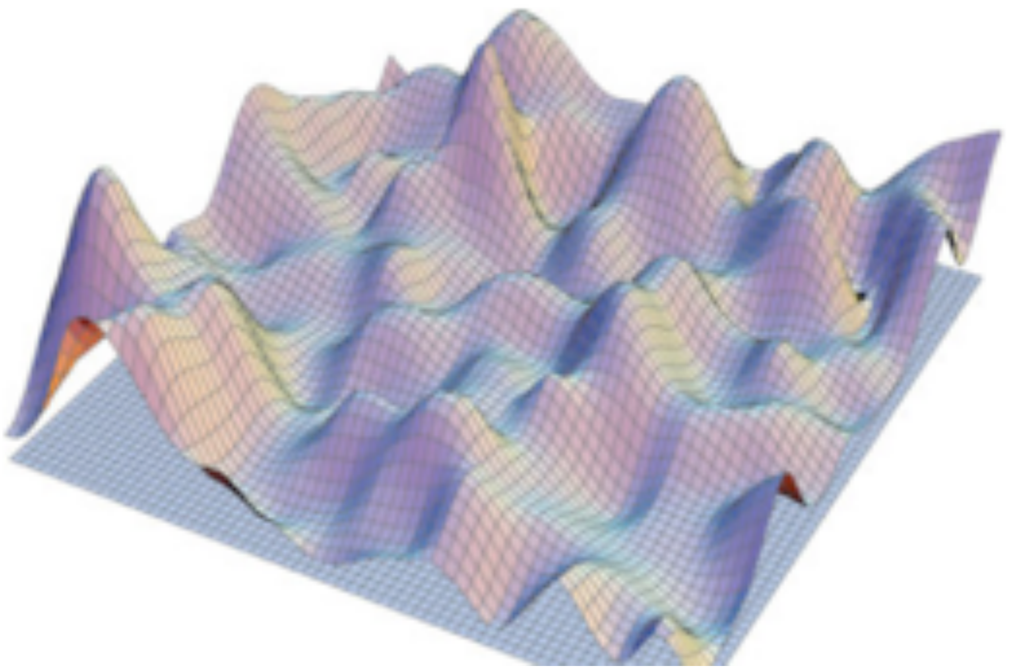
GWs from inflation

Inflation

Accelerated expansion in the early Universe



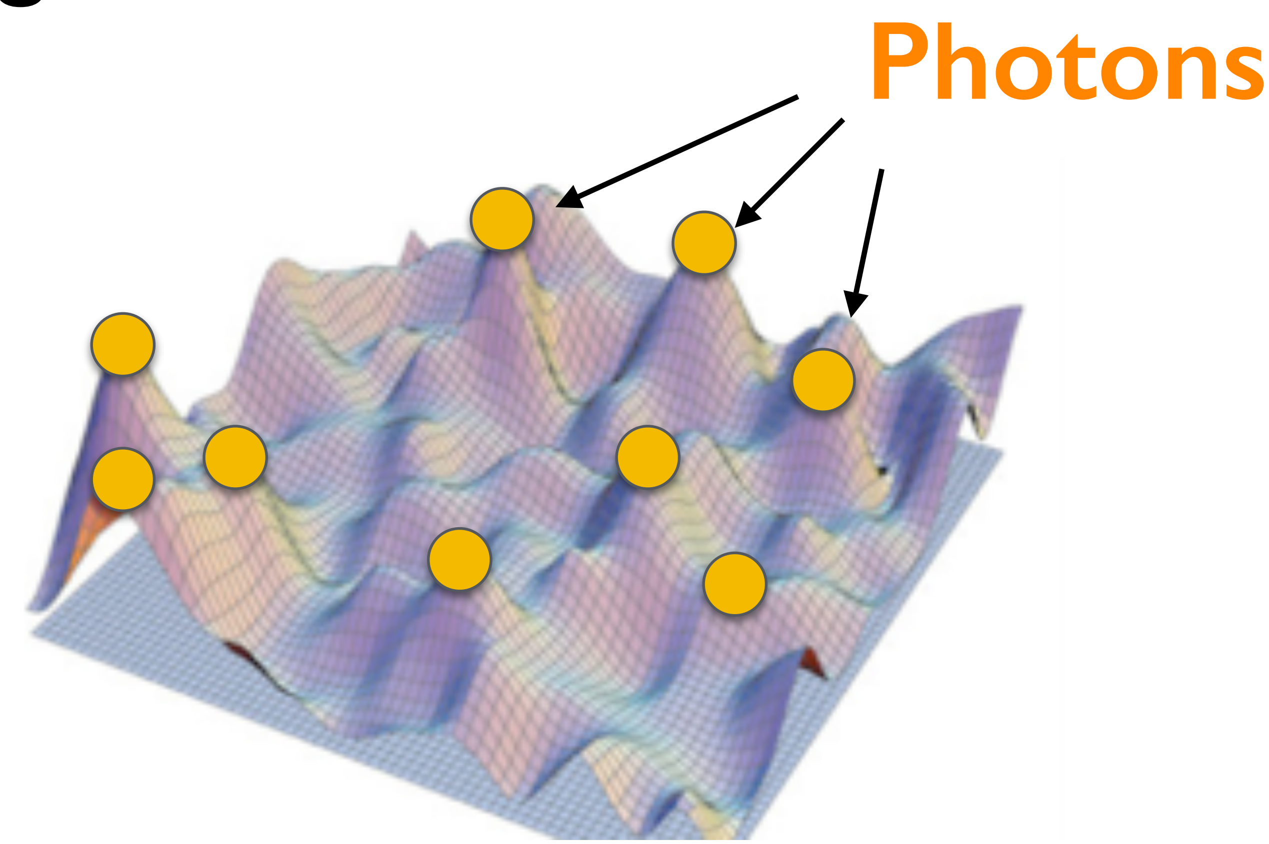
In particle physics…



Particles are excitations in **a field**

In particle physics…

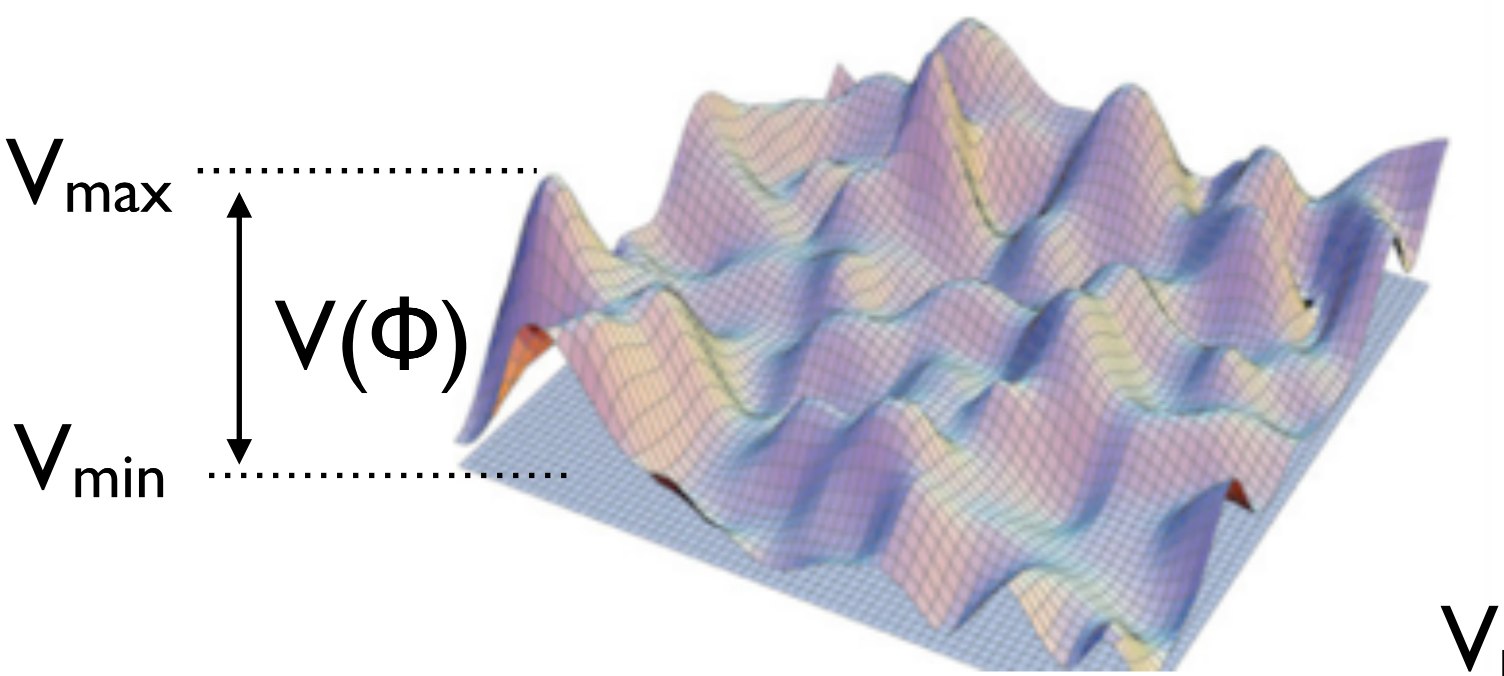
ex: Electromagnetic field

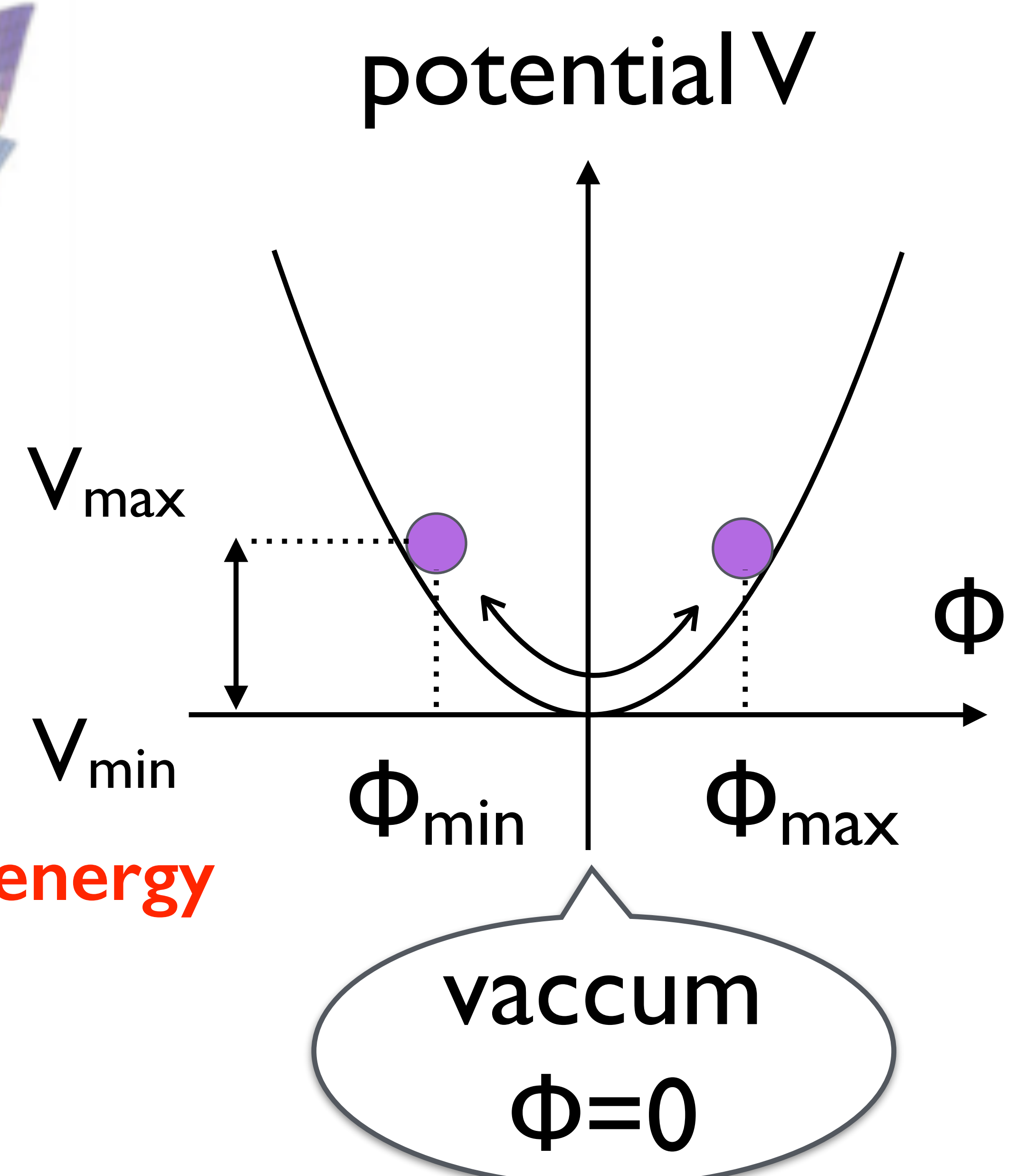


Particles are excitations in **a field**

In particle physics…

Let's consider a scalar field **Φ(x)**





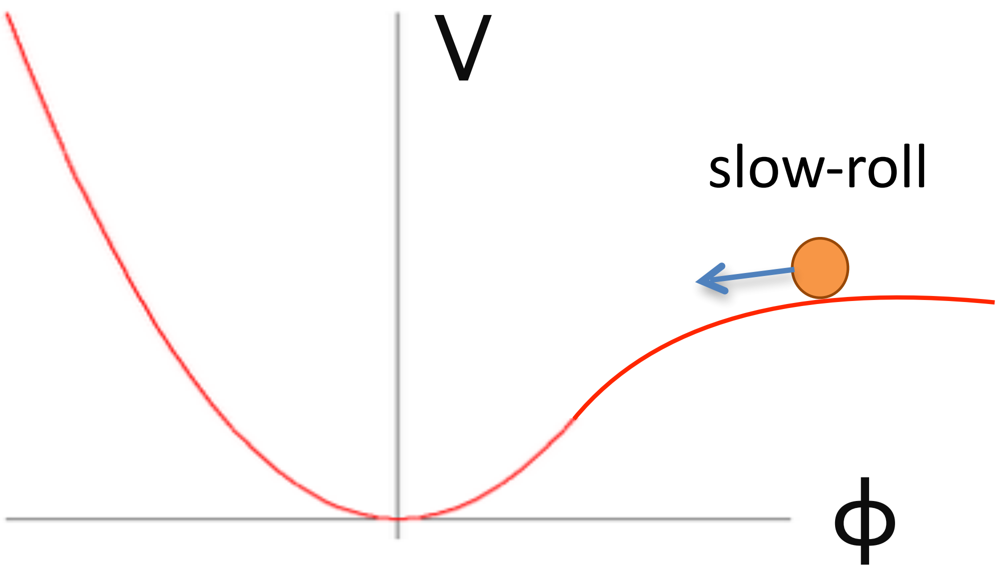
height = energy

What drives inflation?

Standard scenario

Inflation is driven by a scalar field slowly rolling down in its potential

→ new particle? modification of gravity?



energy density of a scalar field

$$\rho_\phi = \dot{\phi}^2/2 + V$$

EOM

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

Friedmann equation

$$H^2 = \frac{8\pi}{3m_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

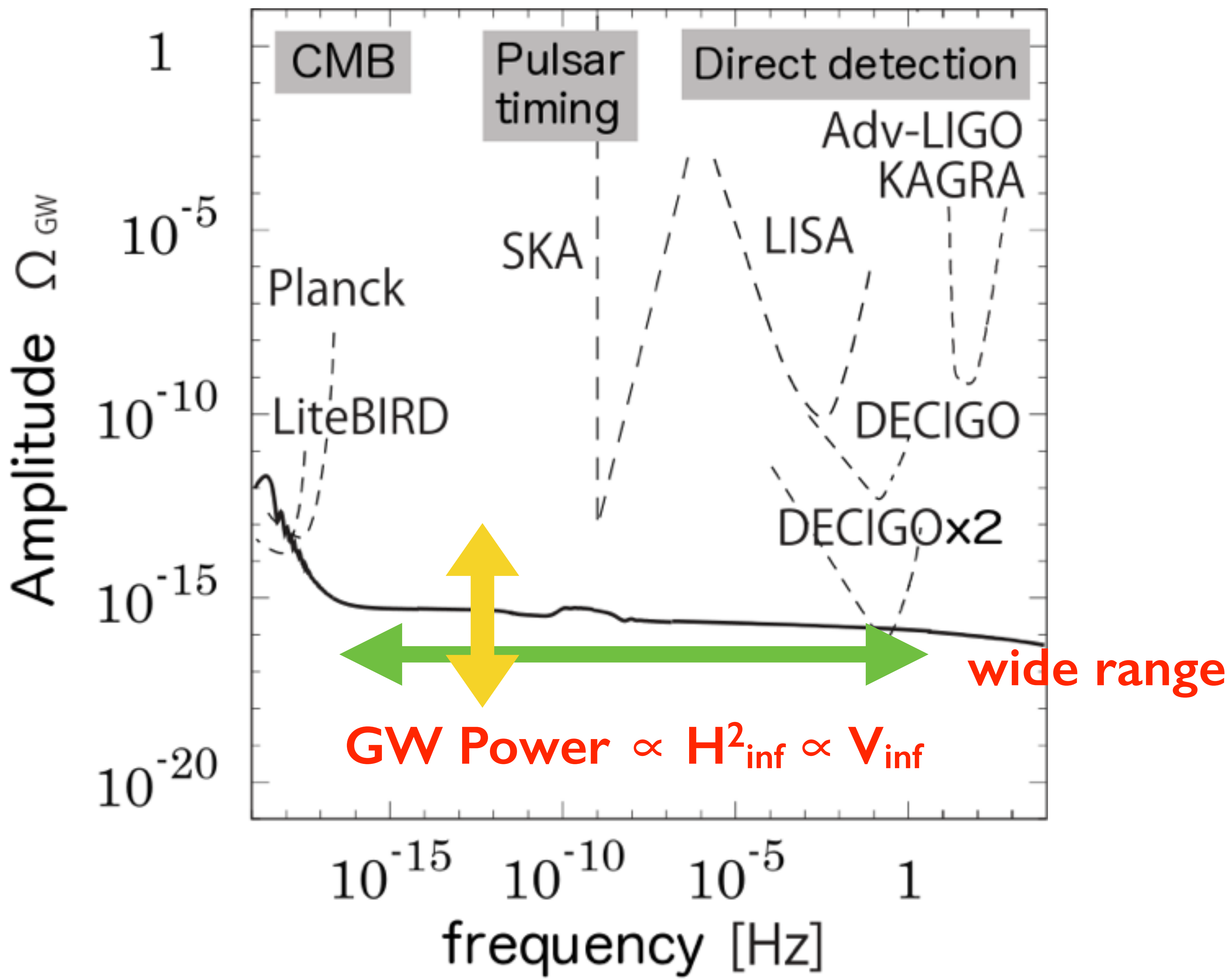
$\dot{\phi}^2/2 \ll V$

$\longrightarrow H = \text{const.}$

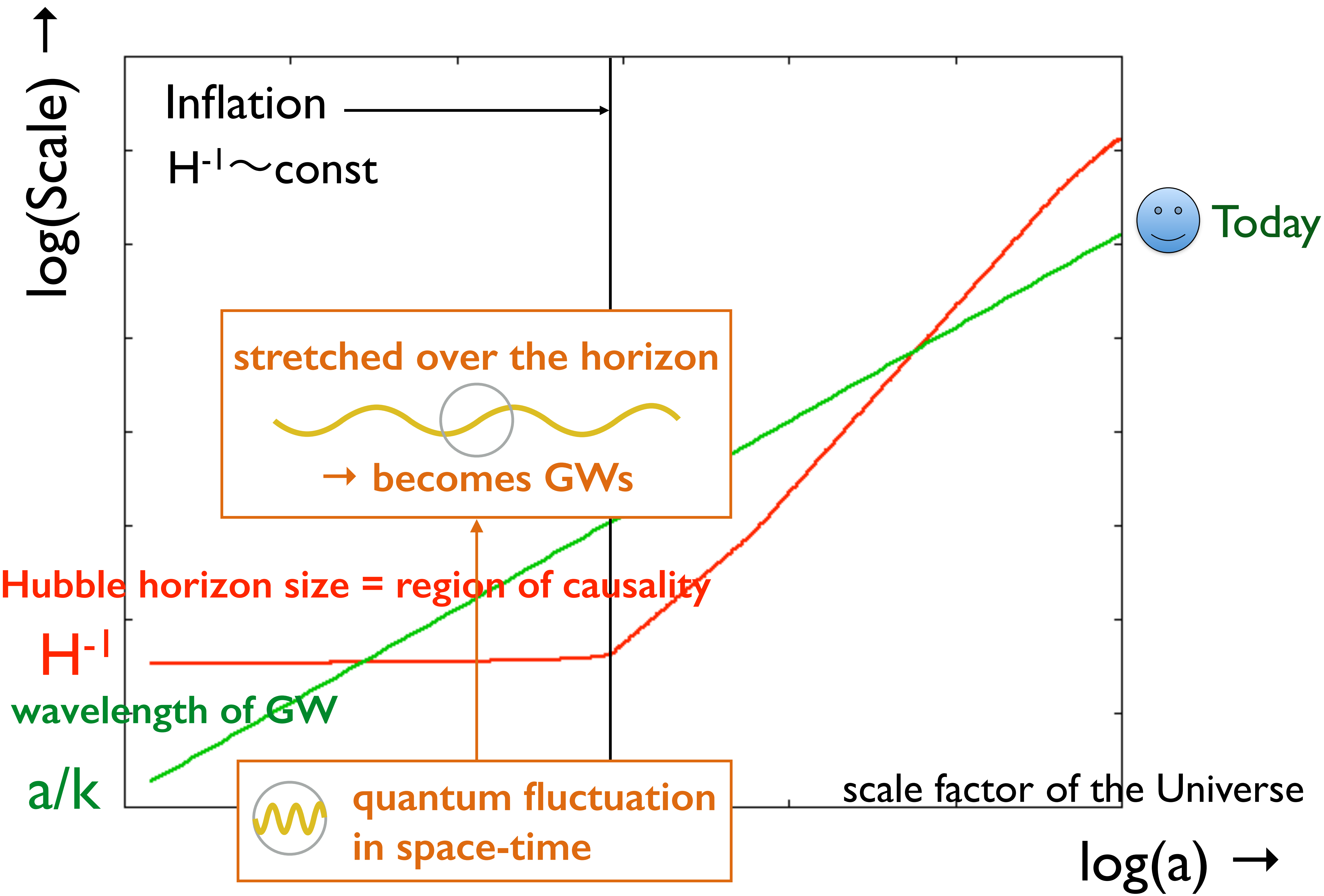
$\to a \propto \exp(Ht)$

Exponential expansion

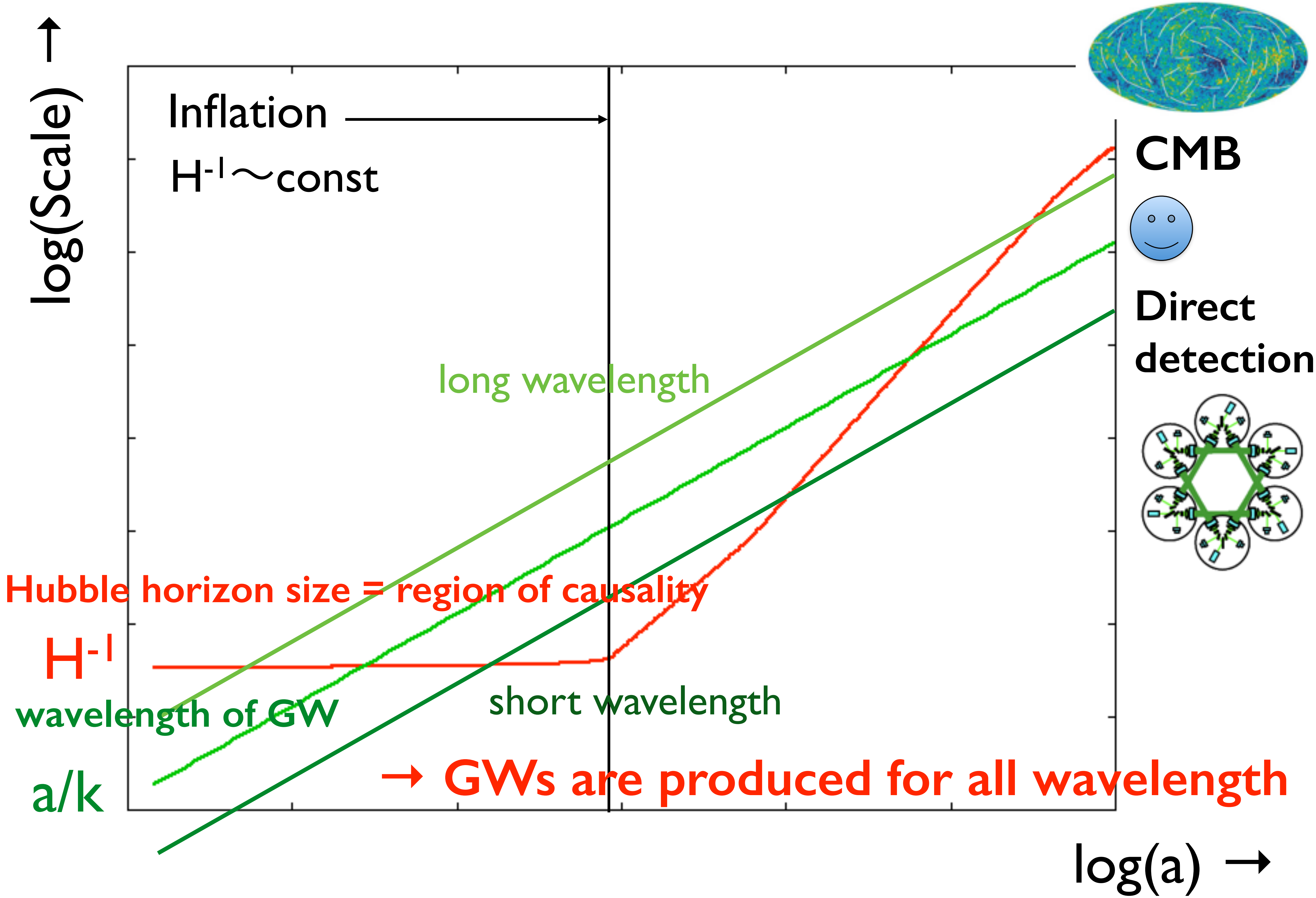
GWs from inflation



Generation mechanism



Generation mechanism



Equation for GWs in the expanding Universe

oscillation

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = 16\pi G \Pi_{ij}$$

expansion of the Universe

effect from matter

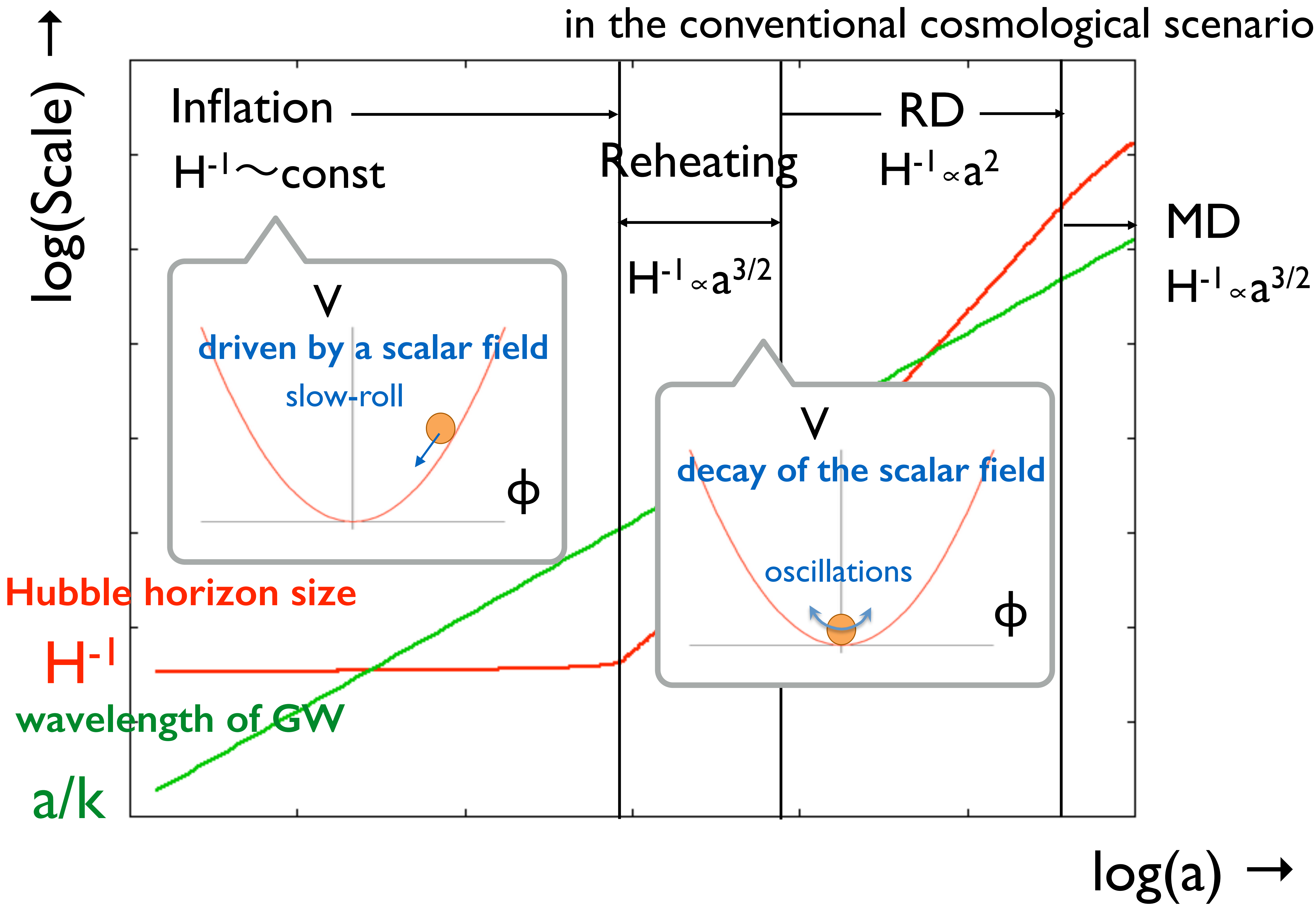
Let us neglect the matter contribution and perform Fourier transform

$$\ddot{h}^{\lambda}_{\mathbf{k}} + \underline{3H\dot{h}^{\lambda}_{\mathbf{k}}} + \underline{\frac{k^2}{a^2}}h^{\lambda}_{\mathbf{k}} = 0$$

- H>k/a $\quad h^{\lambda}_{\mathbf{k}} \propto \mathrm{const.}$
- H<k/a $\quad h^{\lambda}_{\mathbf{k}} \propto a^{-1}e^{-ik\tau}.$

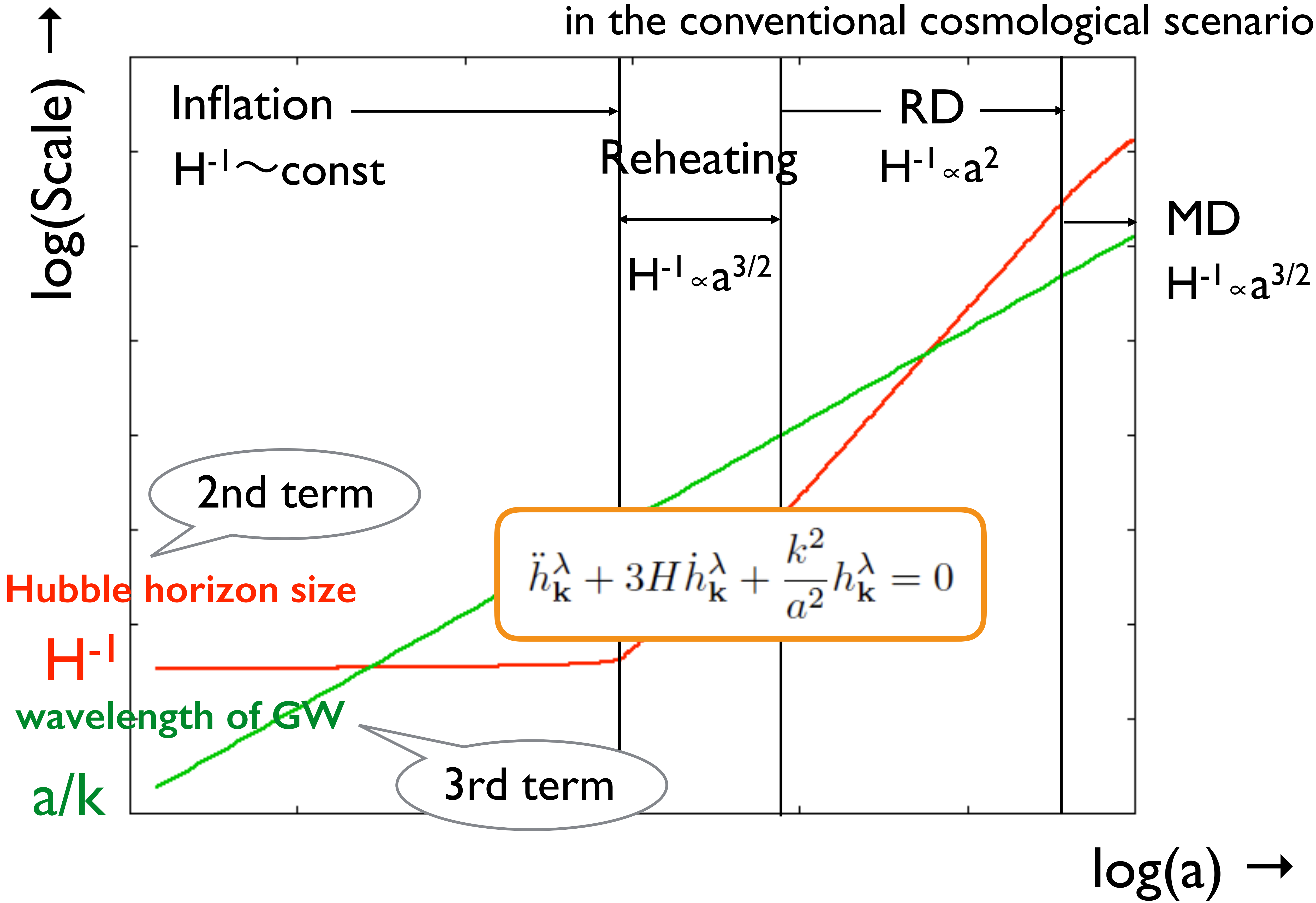
→ behavior is determined by the balance between H (Hubble) and k (wavenumber = f/2π)

Hubble expansion history after inflation

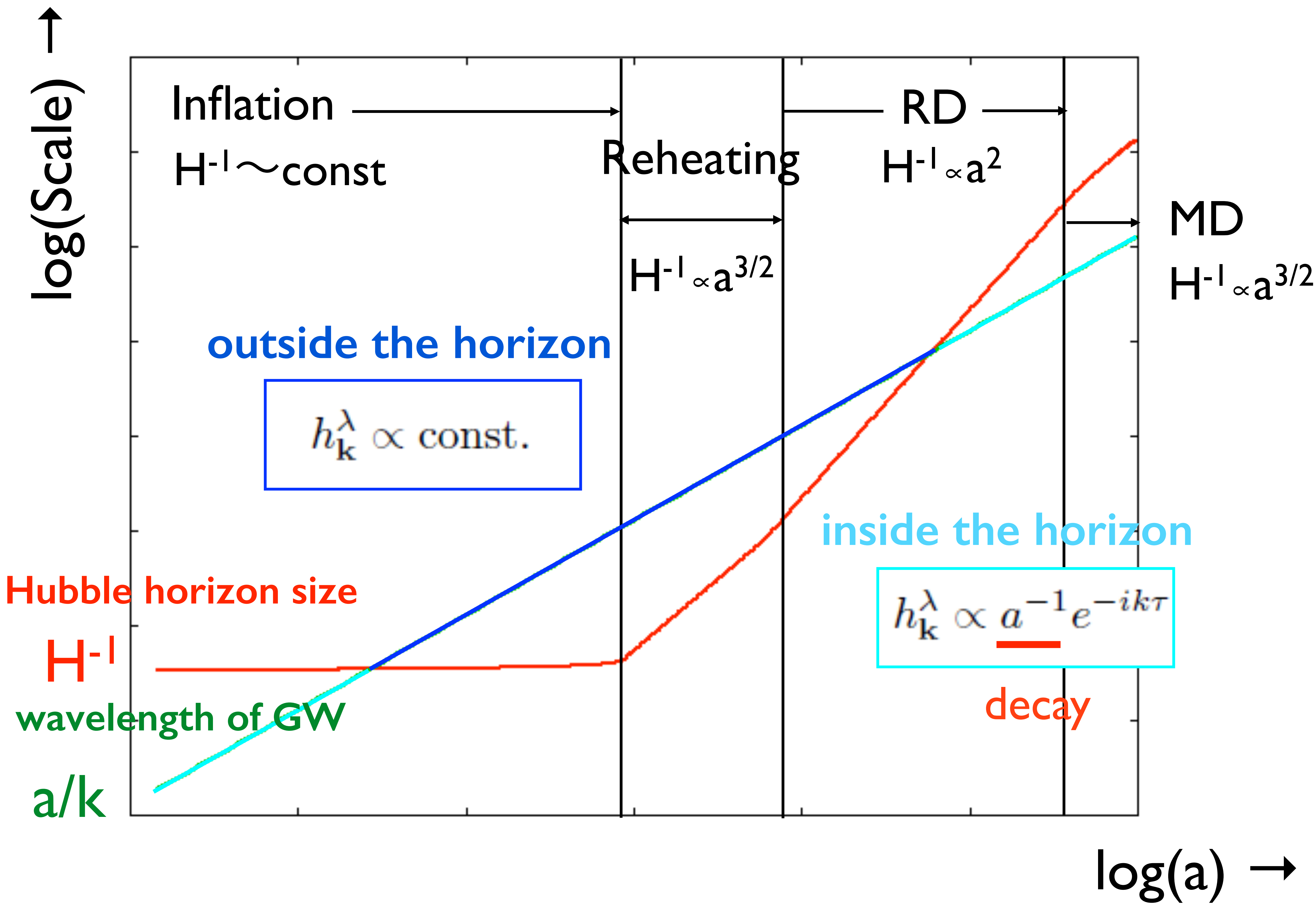


Hubble expansion history after inflation

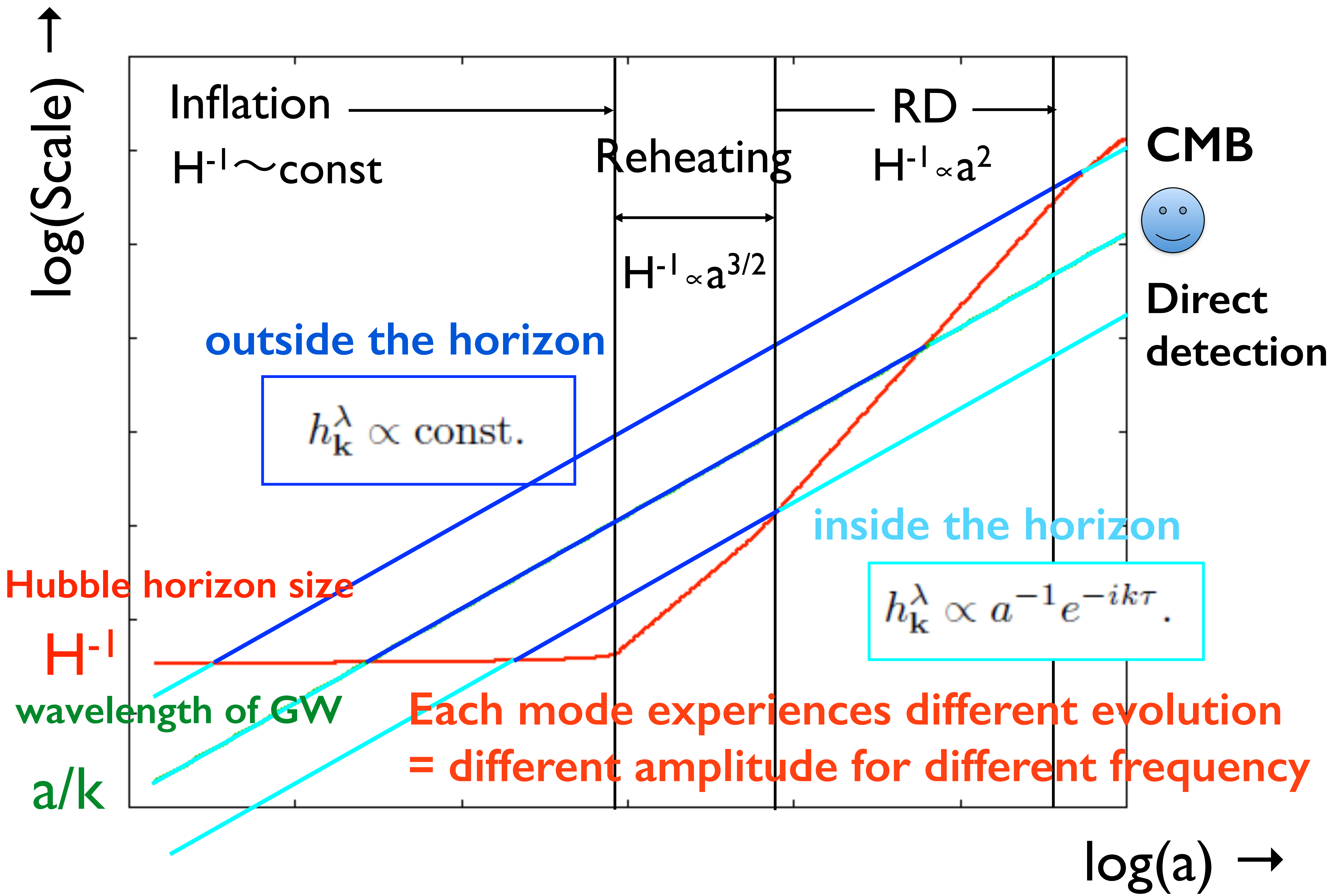
in the conventional cosmological scenario



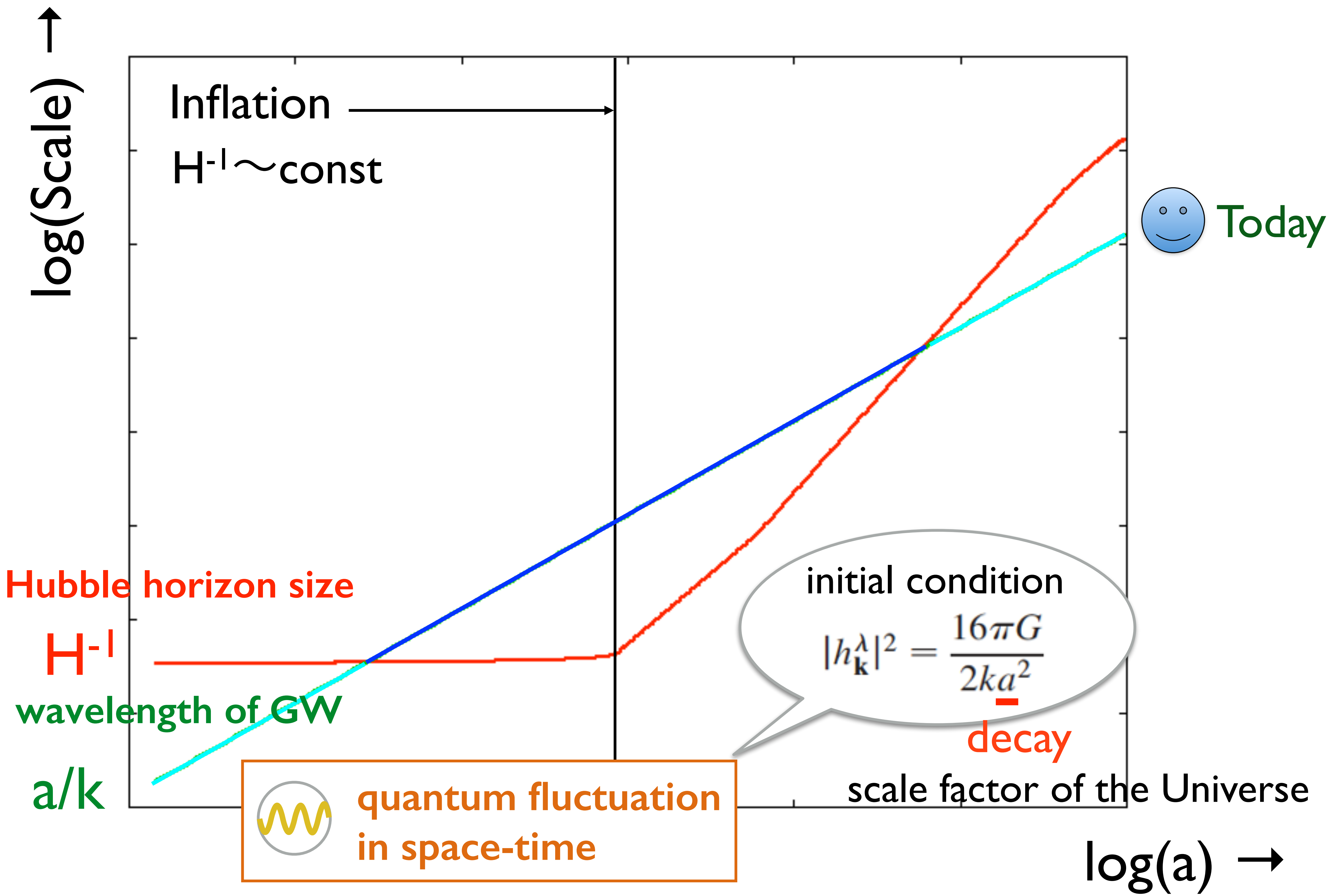
Evolution of GWs



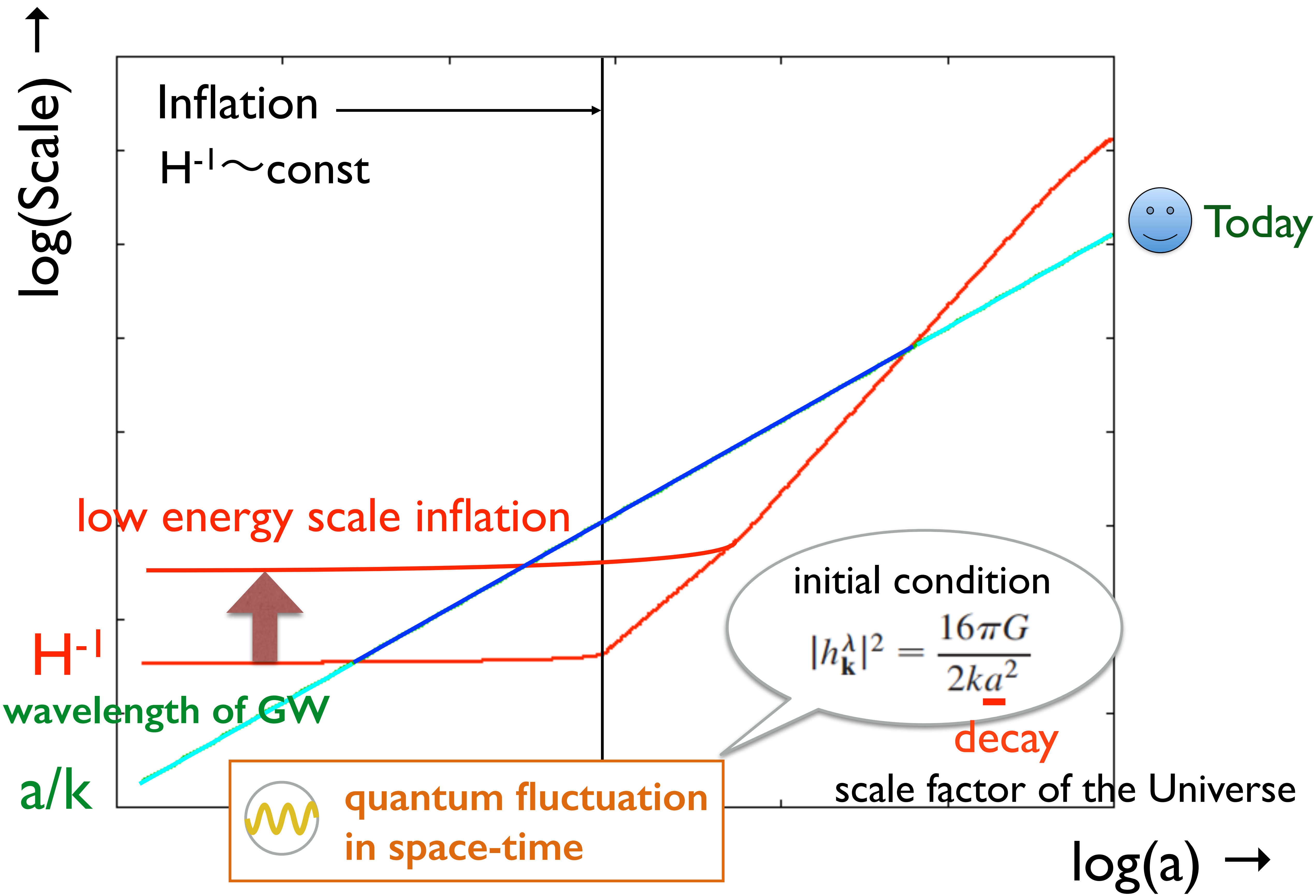
Hubble expansion history after inflation



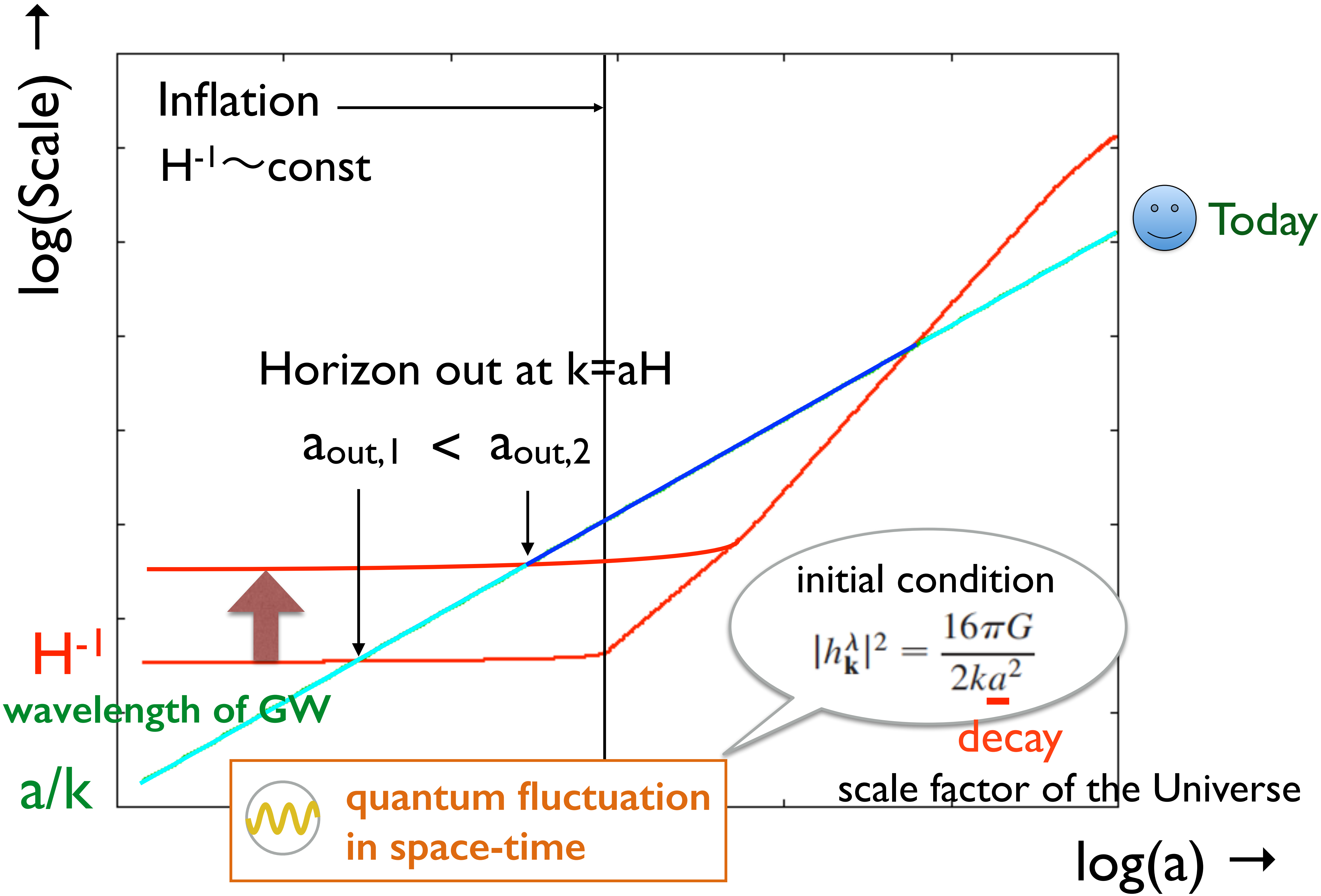
GW amplitude



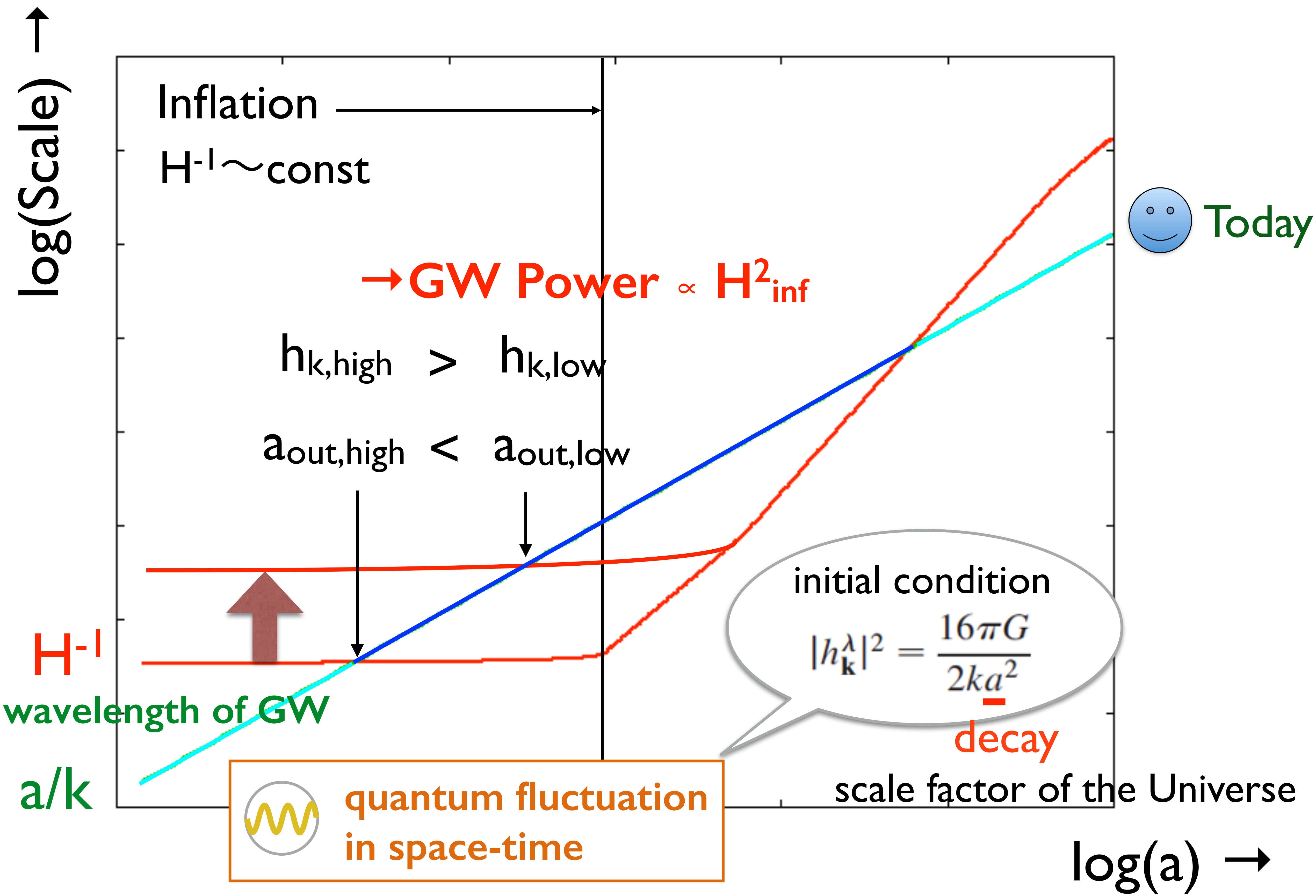
GW amplitude



GW amplitude



GW amplitude



Spectral shape

Amplitude of GW background $\Omega_{GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln k} = \frac{1}{12}\left(\frac{k}{aH}\right)^2 \frac{k^3}{\pi^2}\sum_\lambda |h_{\mathbf{k}}^\lambda|^2$

Outside the horizon $h_{\mathbf{k}}^\lambda \propto \text{const.}$

Inside the horizon $h_{\mathbf{k}}^\lambda \propto a^{-1}e^{-ik\tau}$

Inflation $a \propto \exp(Ht)$

scale invariant spectrum

reheating $a \propto t^{2/3} \longrightarrow k^{-2}$

small scale modes begin to enter the horizon and damp with $\propto a^{-1}$

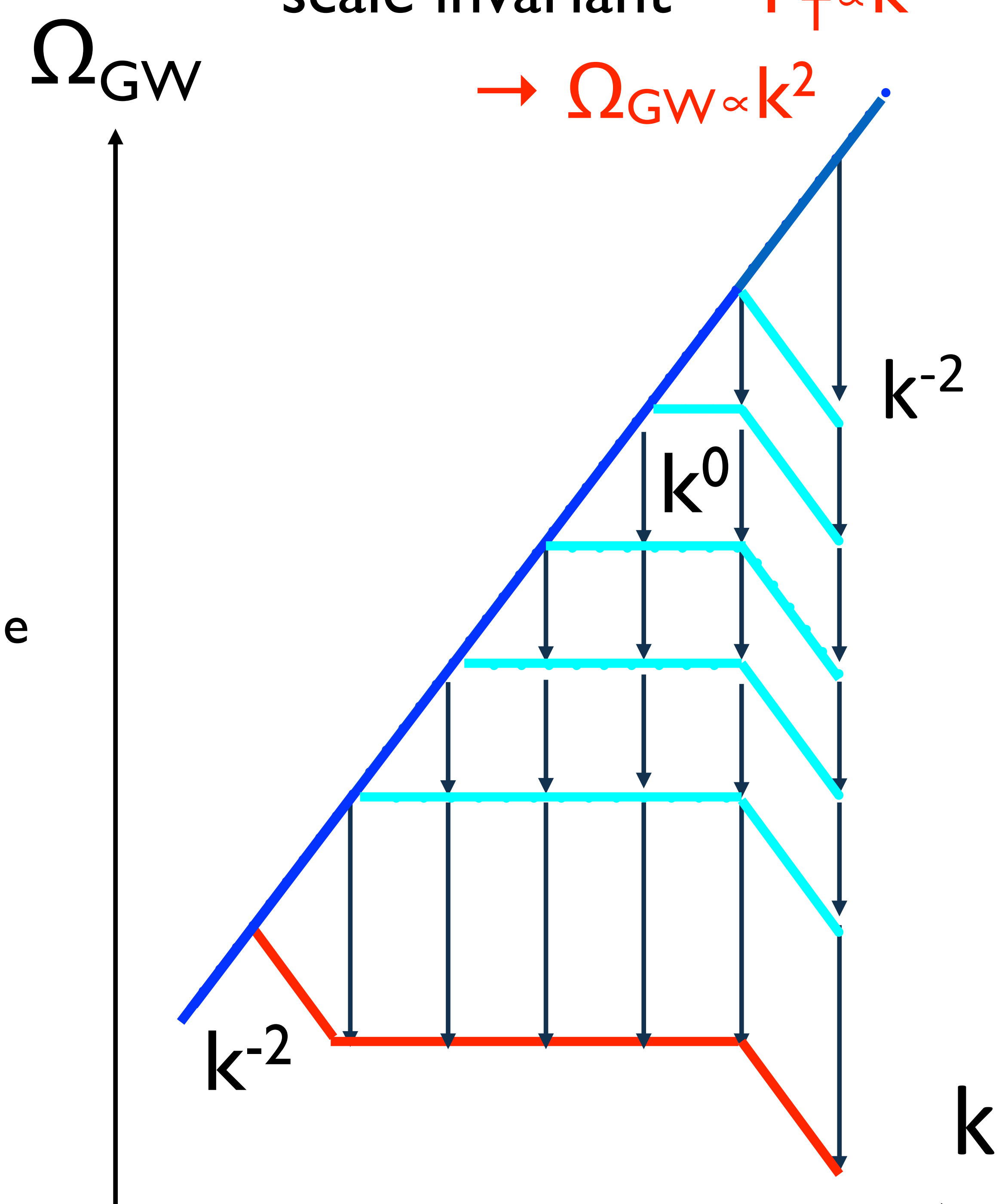
radiation dominant $a \propto t^{1/2} \longrightarrow k^{0}$

the expansion decelerates
= damping $\propto a^{-1}$ becomes smaller

matter dominant $a \propto t^{2/3} \longrightarrow k^{-2}$

scale invariant = $P_T \propto k^0$

$\rightarrow \Omega_{GW} \propto k^2$



Spectral shape

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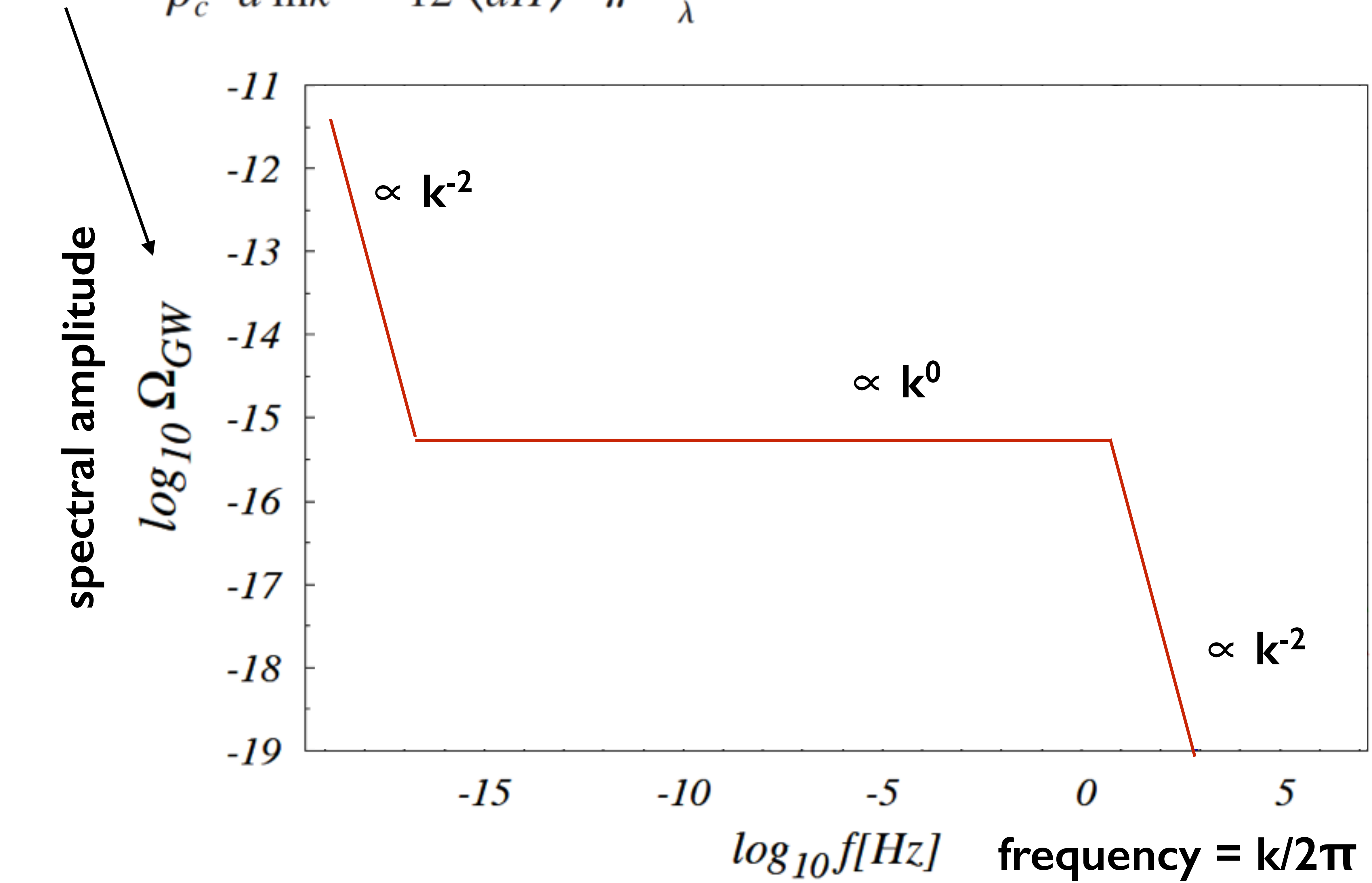

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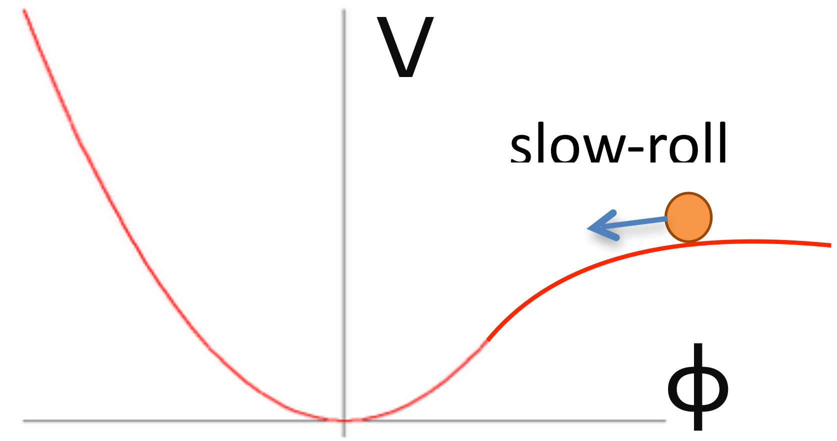
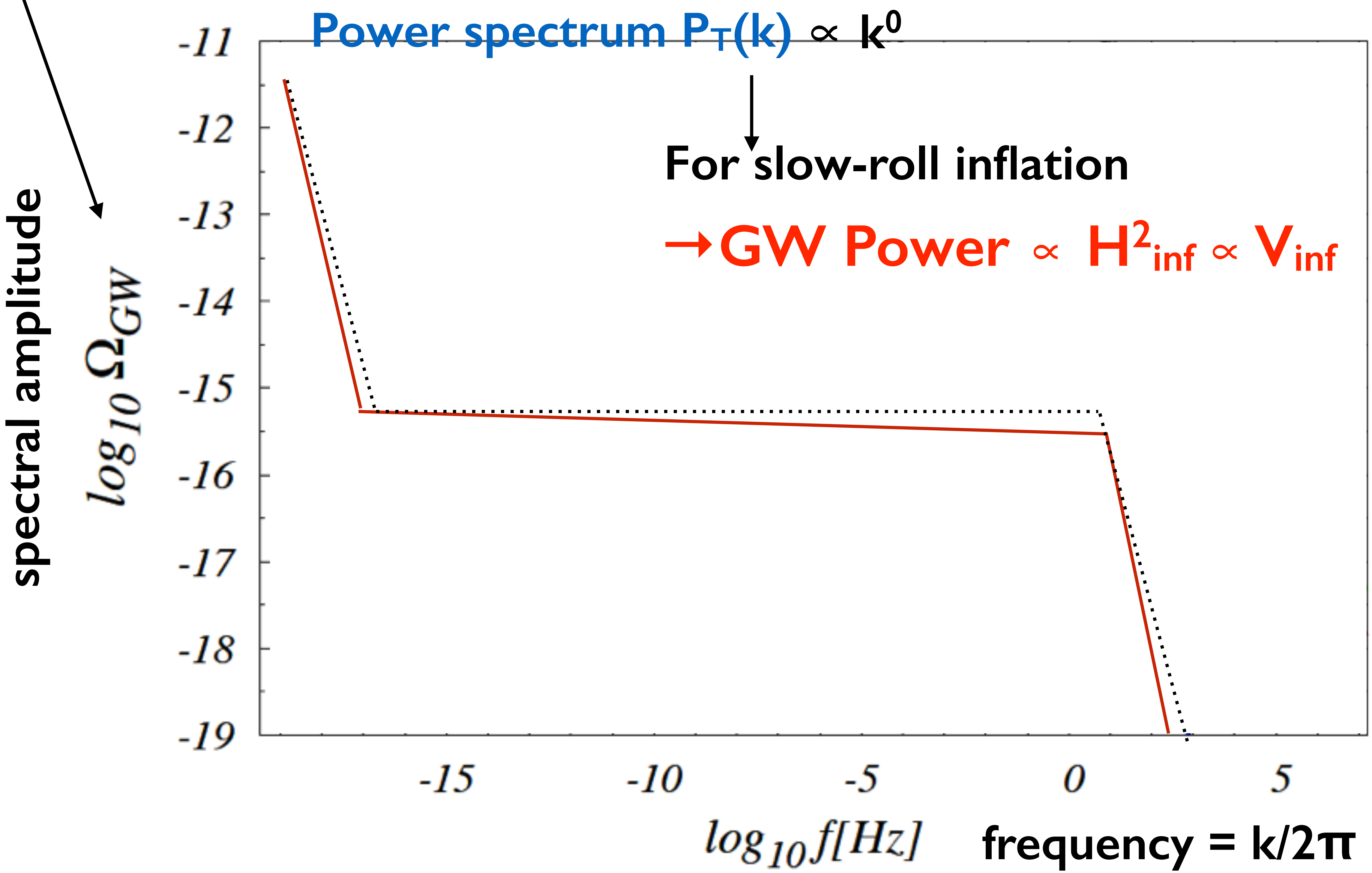


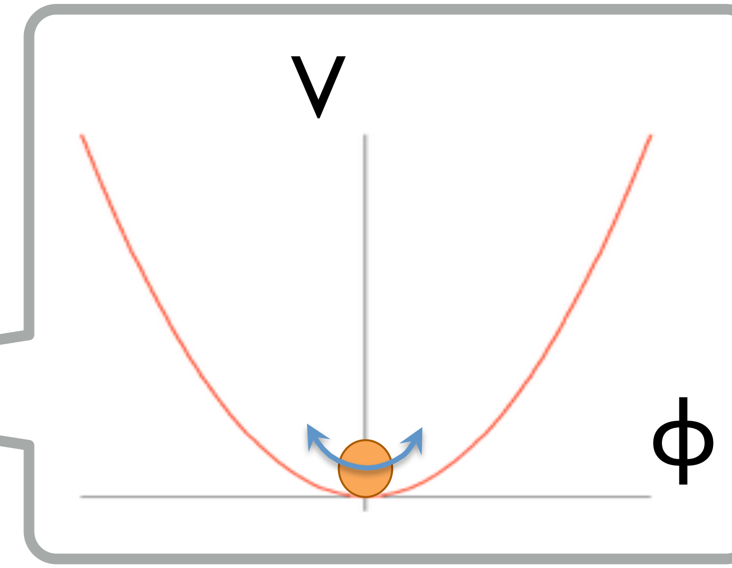


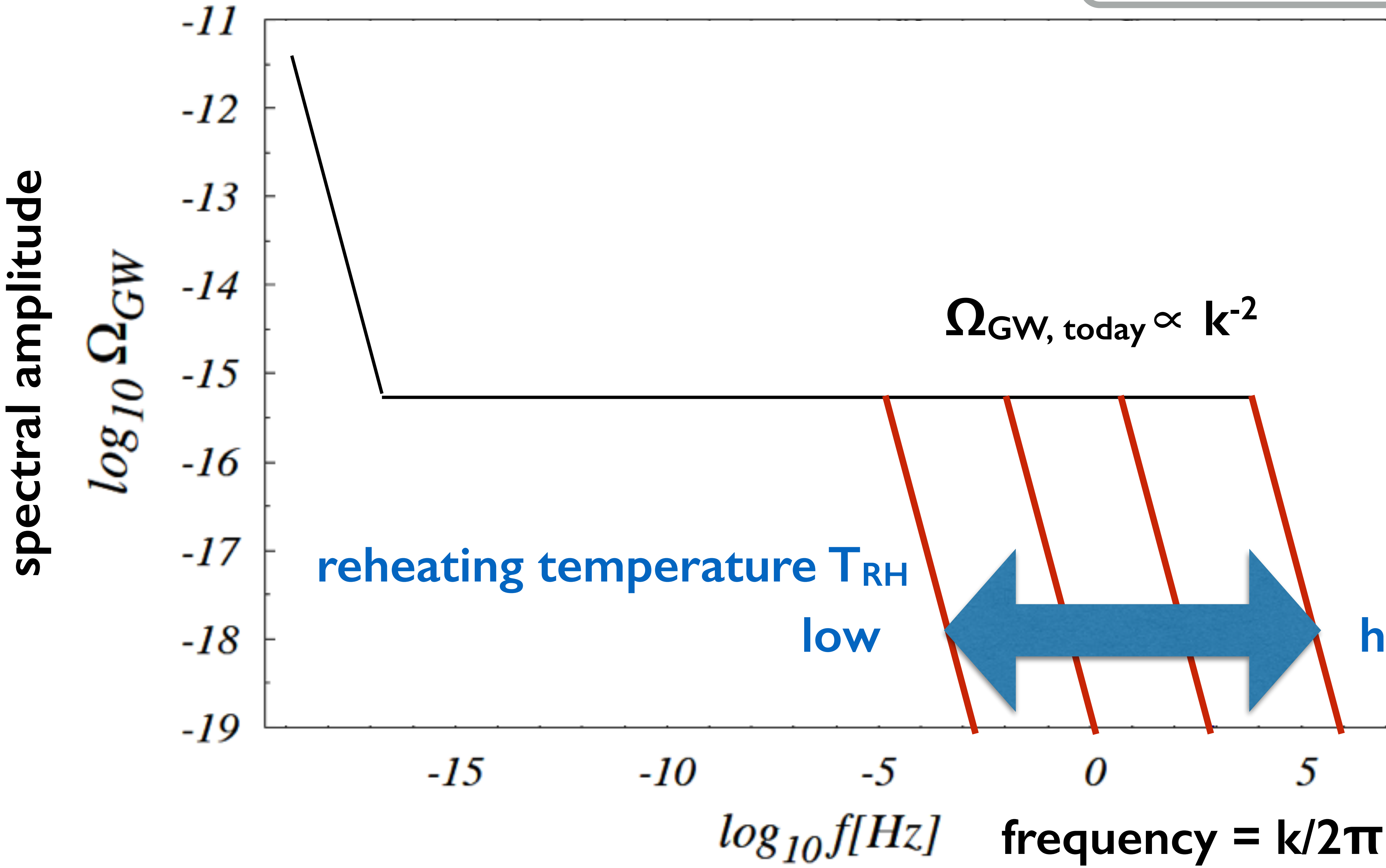
Effect of reheating

Reheating after inflation

for Φ^2 potential, $H \propto a^{-3/2}$

→ matter-dominated Universe





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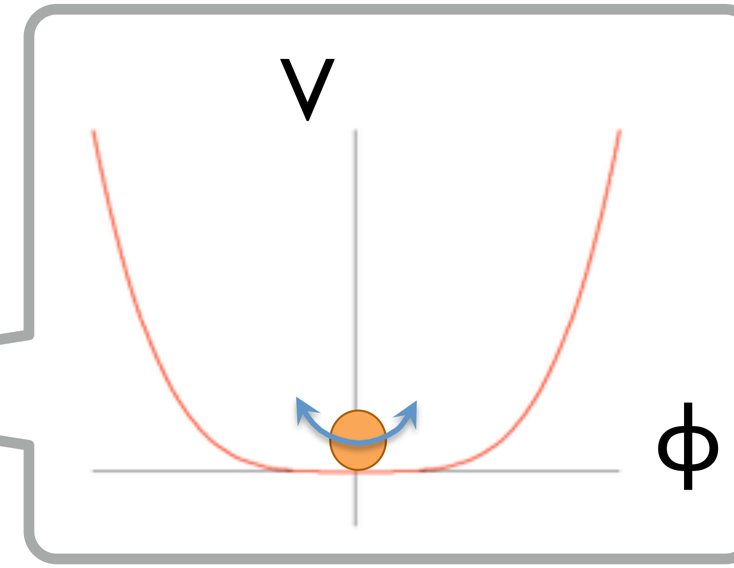
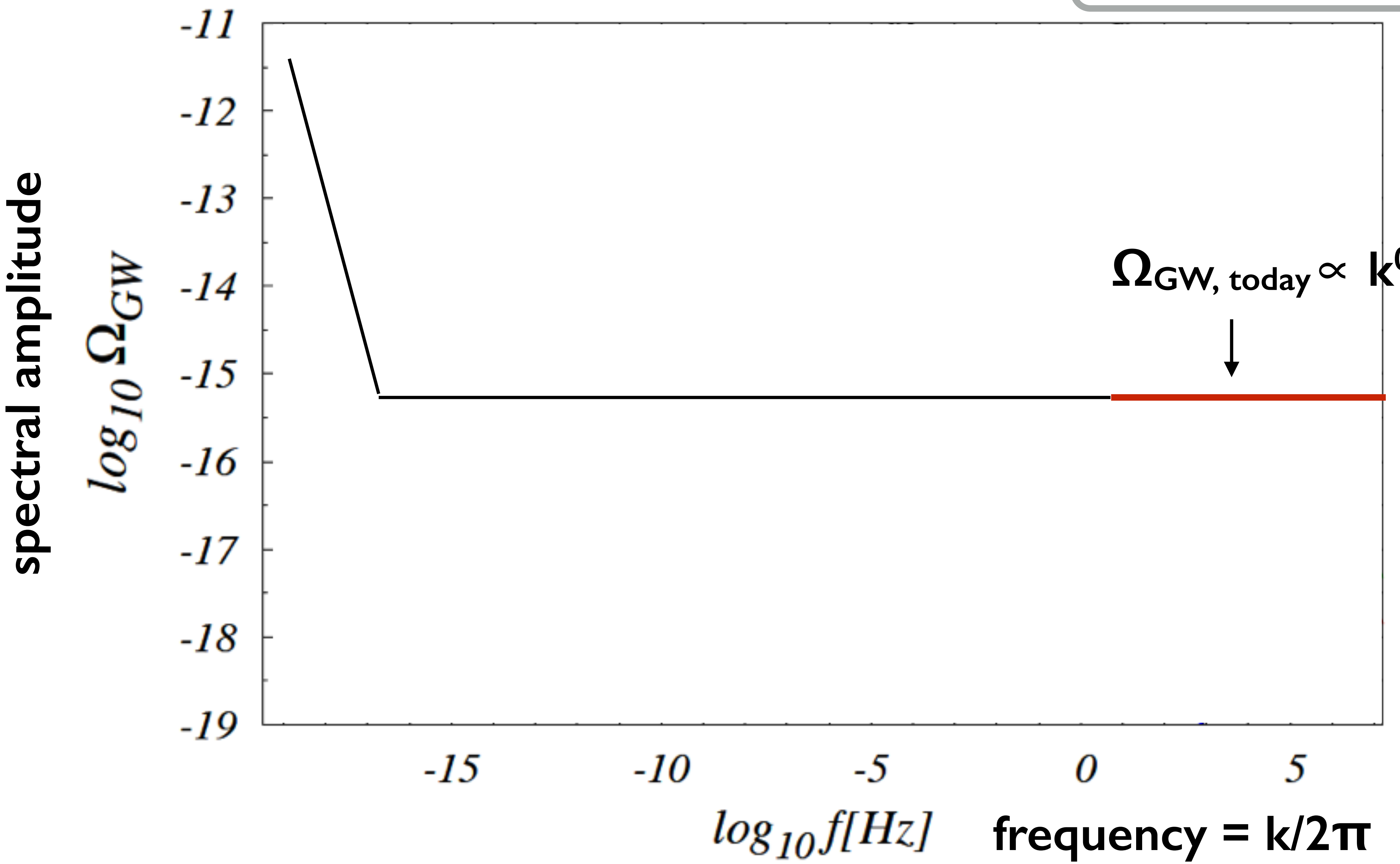


Reheating after inflation

for Φ^4 potential, $H \propto a^{-2}$

→ radiation-dominated Universe



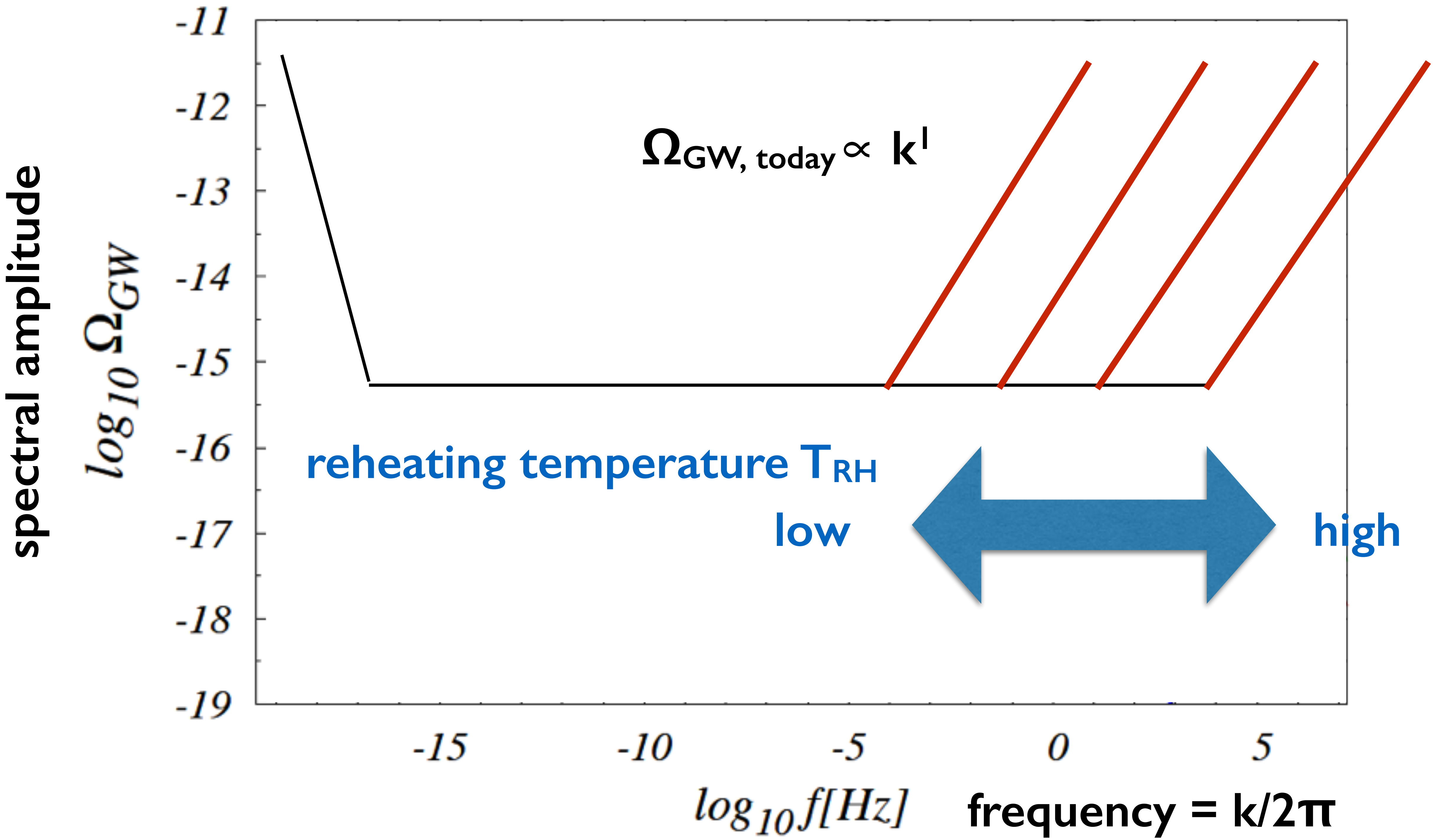
Effect of reheating

$\rho_\phi = \dot{\phi}^2/2 + V$

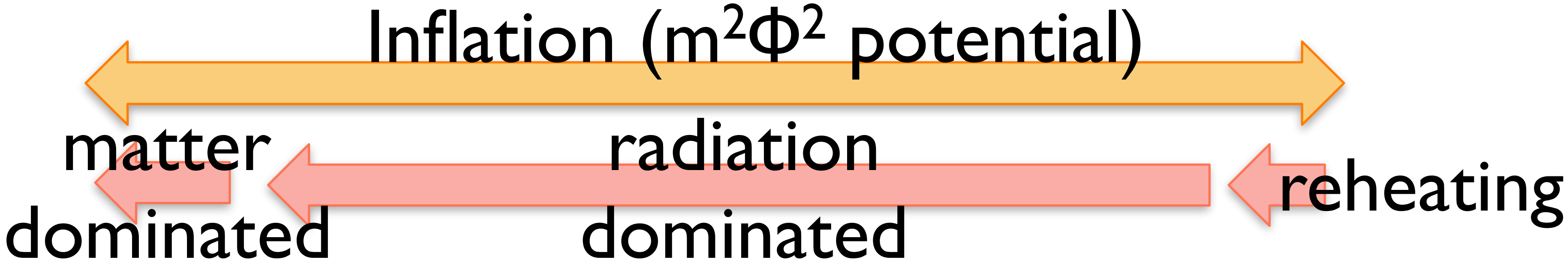
Reheating after inflation

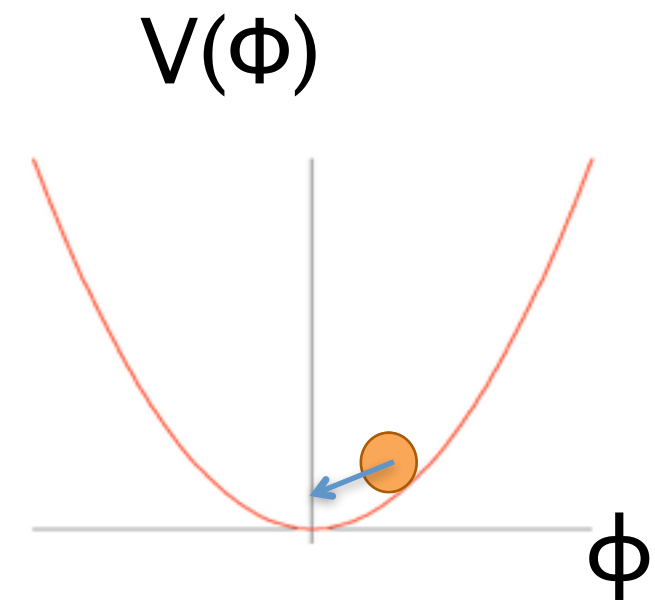
kinatic energy > potential energy → H $\propto a^{-6}$

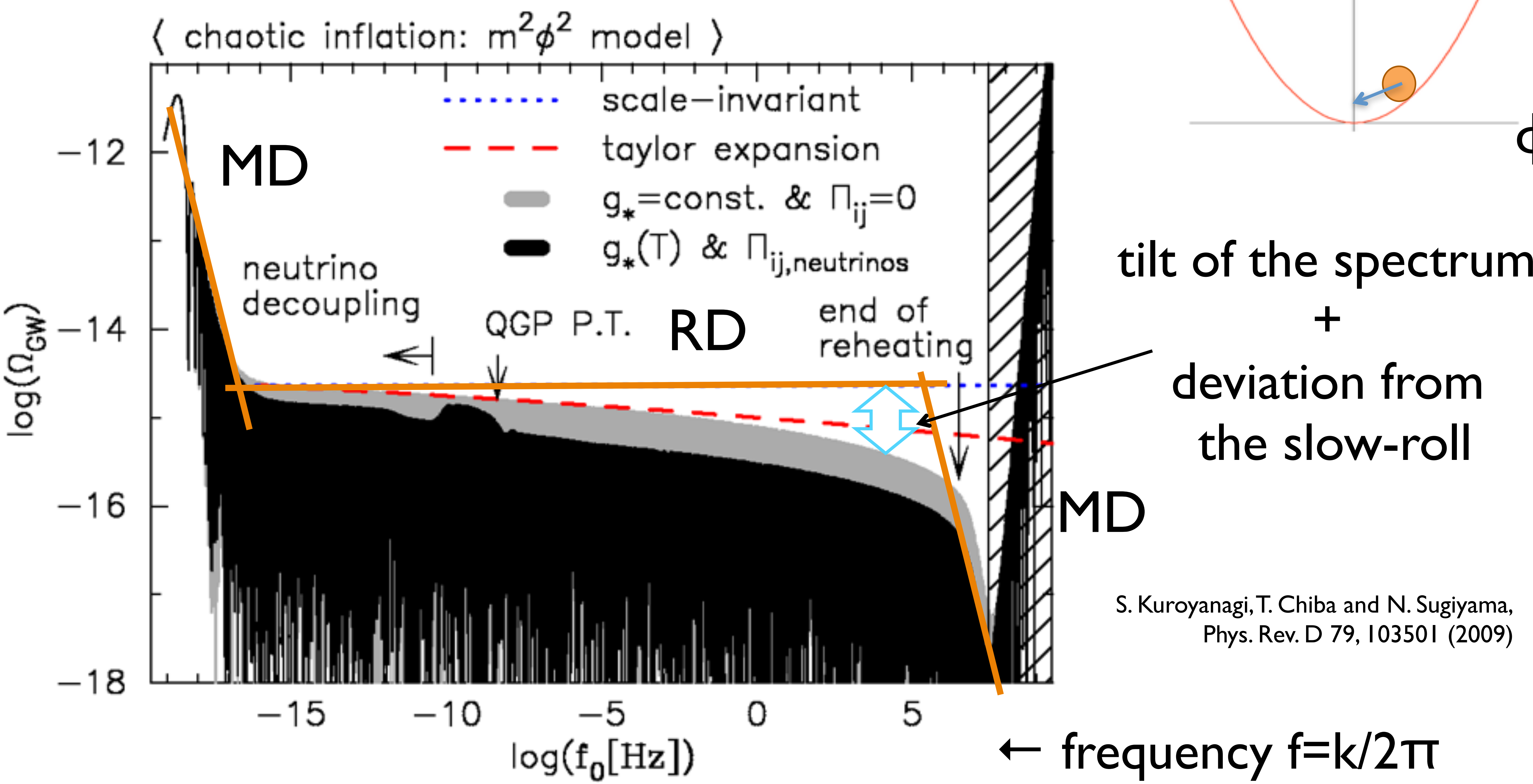
→ kination-dominated Universe



Spectral shape from numerical calculation



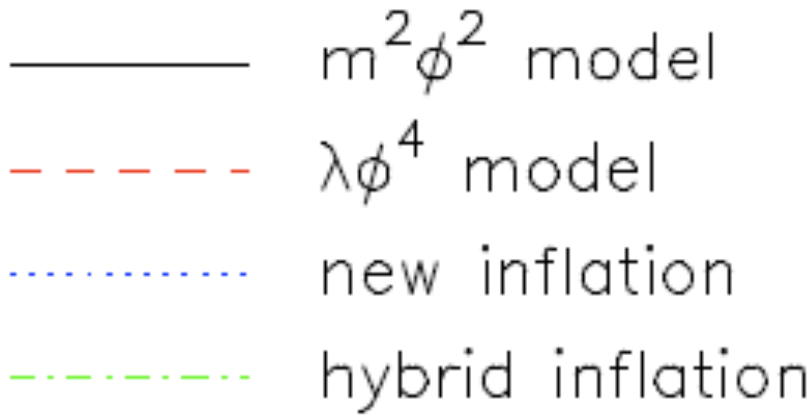


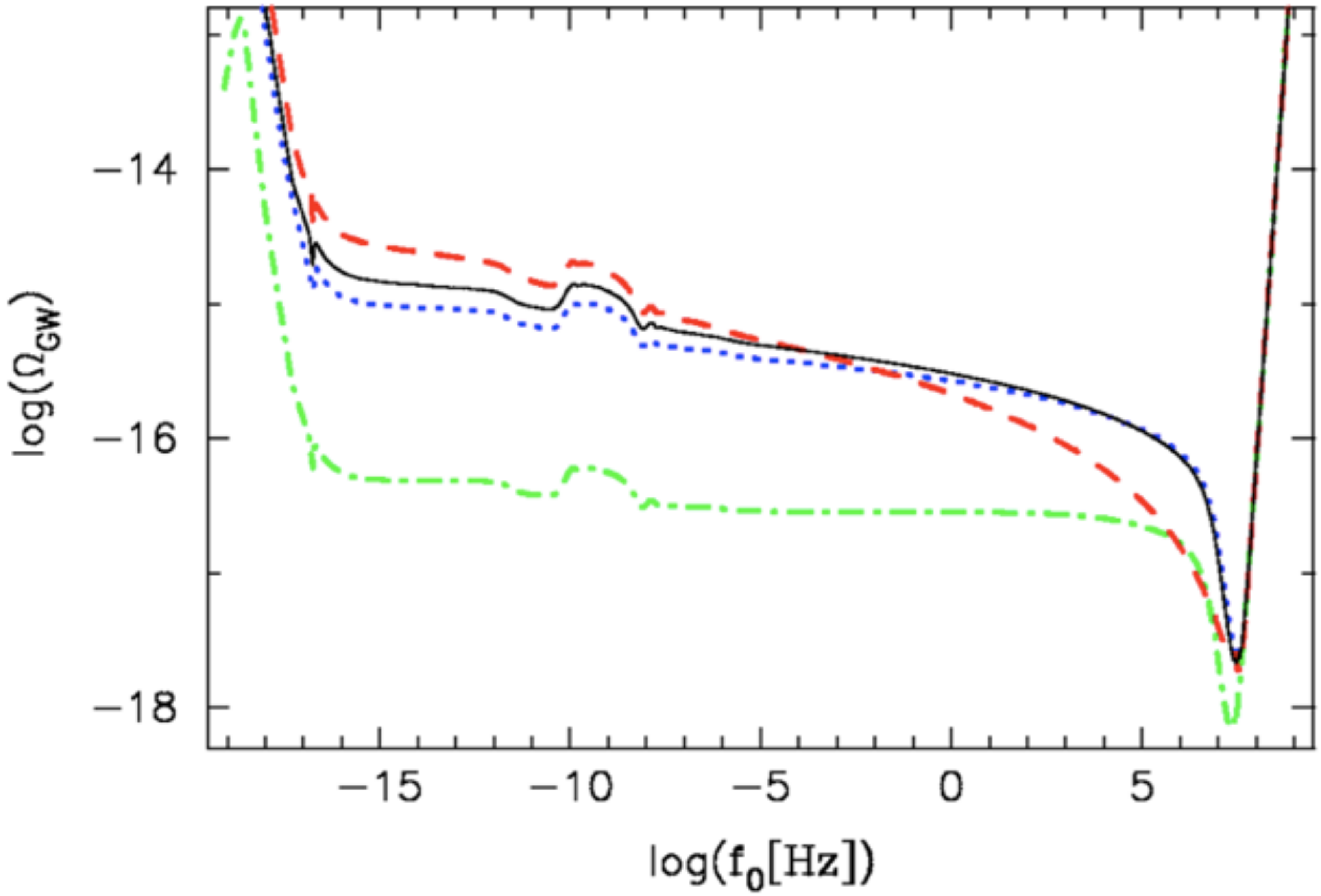


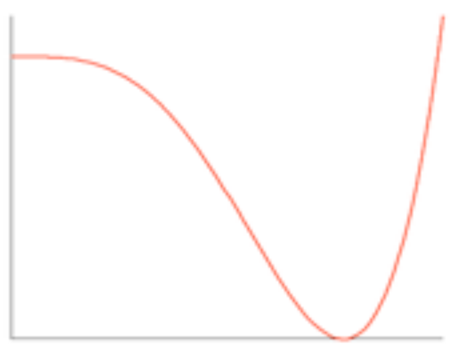
S. Kuroyanagi, T. Chiba and N. Sugiyama, Phys. Rev. D 79, 103501 (2009)

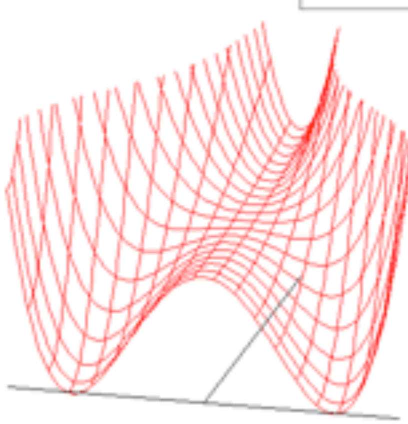
Other inflation models

Kuroyanagi et al. PRD 79, 103501 (2009)







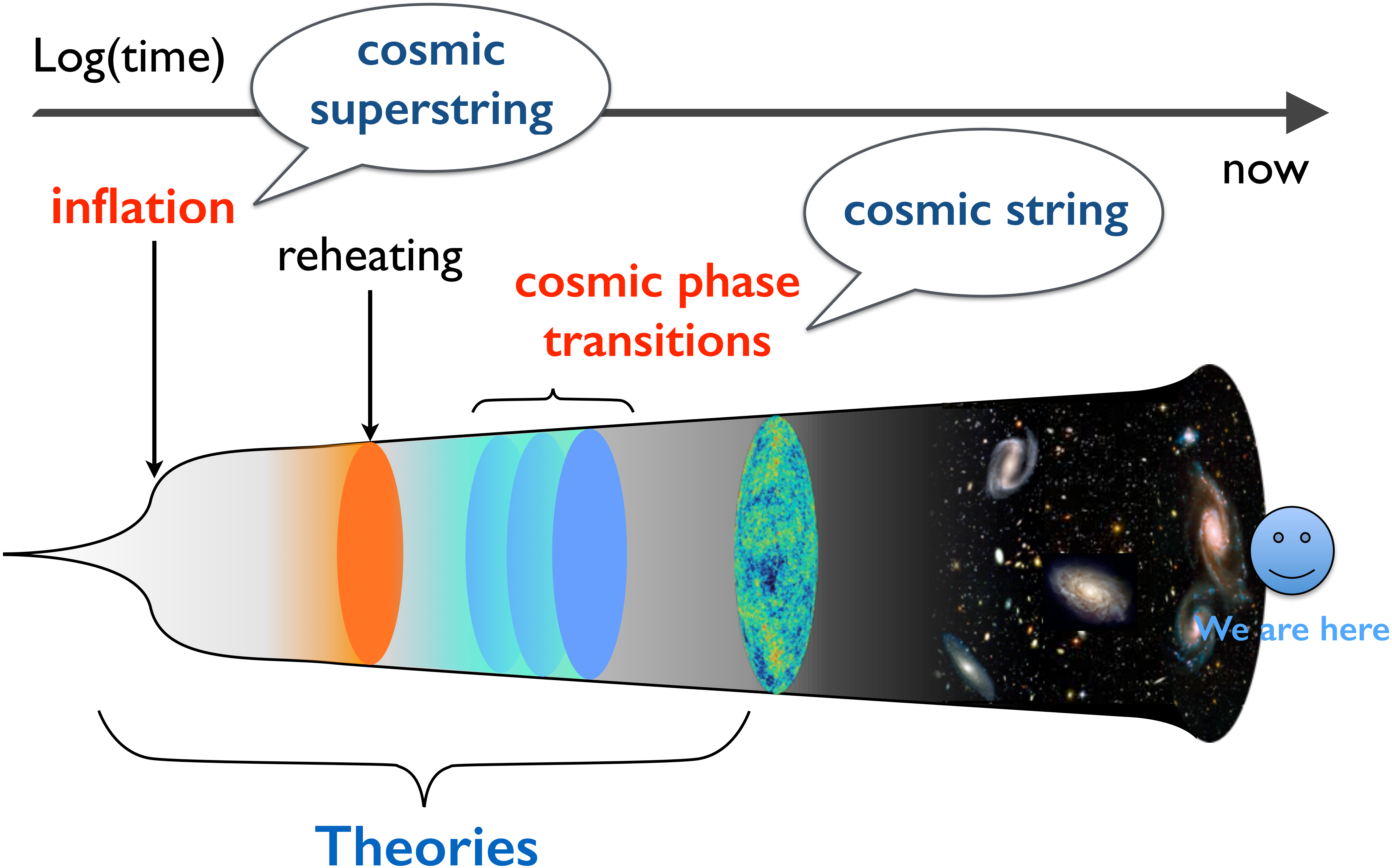


→ Differences in the amplitude and the tilt

→ can be used to specify inflation model

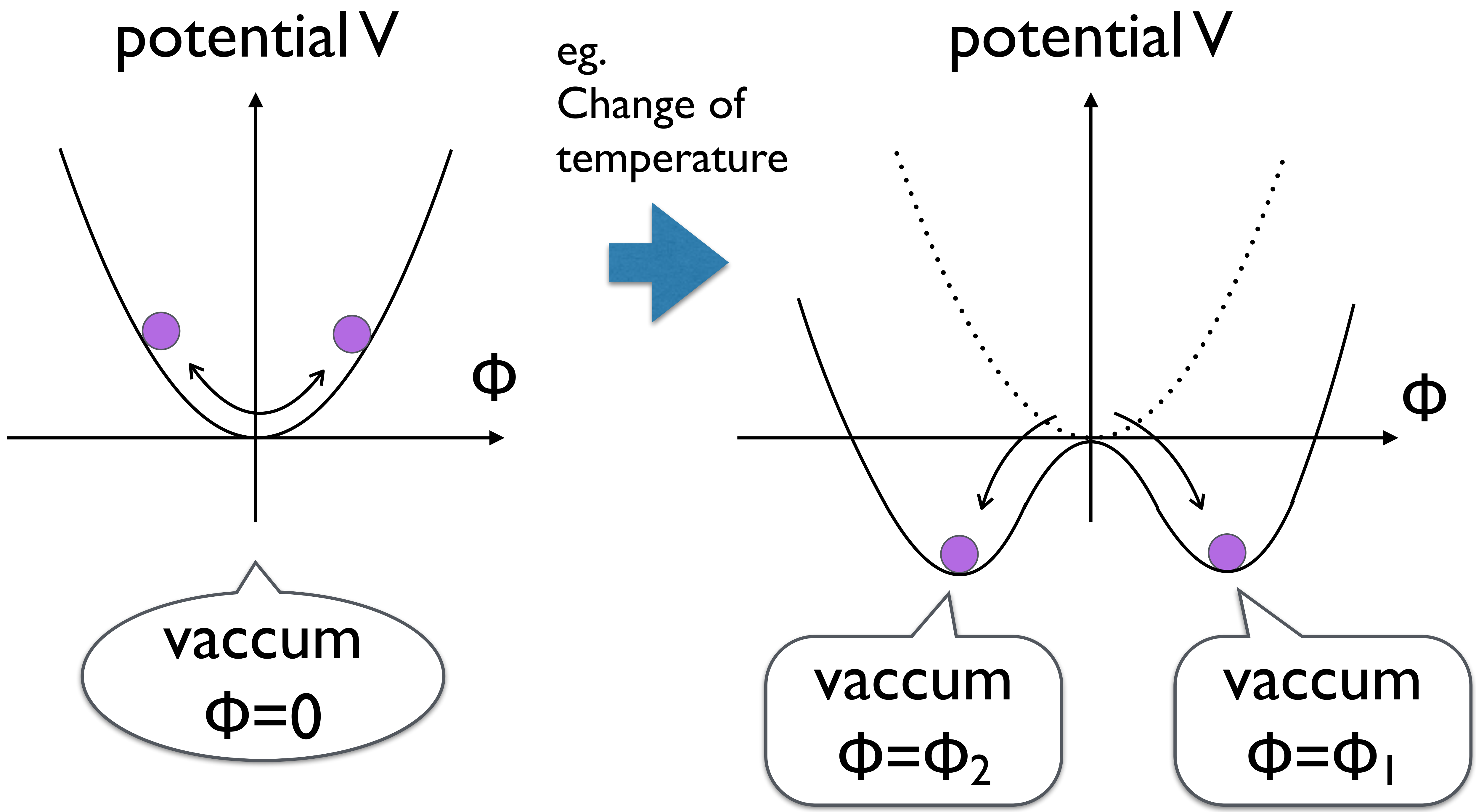
GWs from cosmic strings

Cosmological history

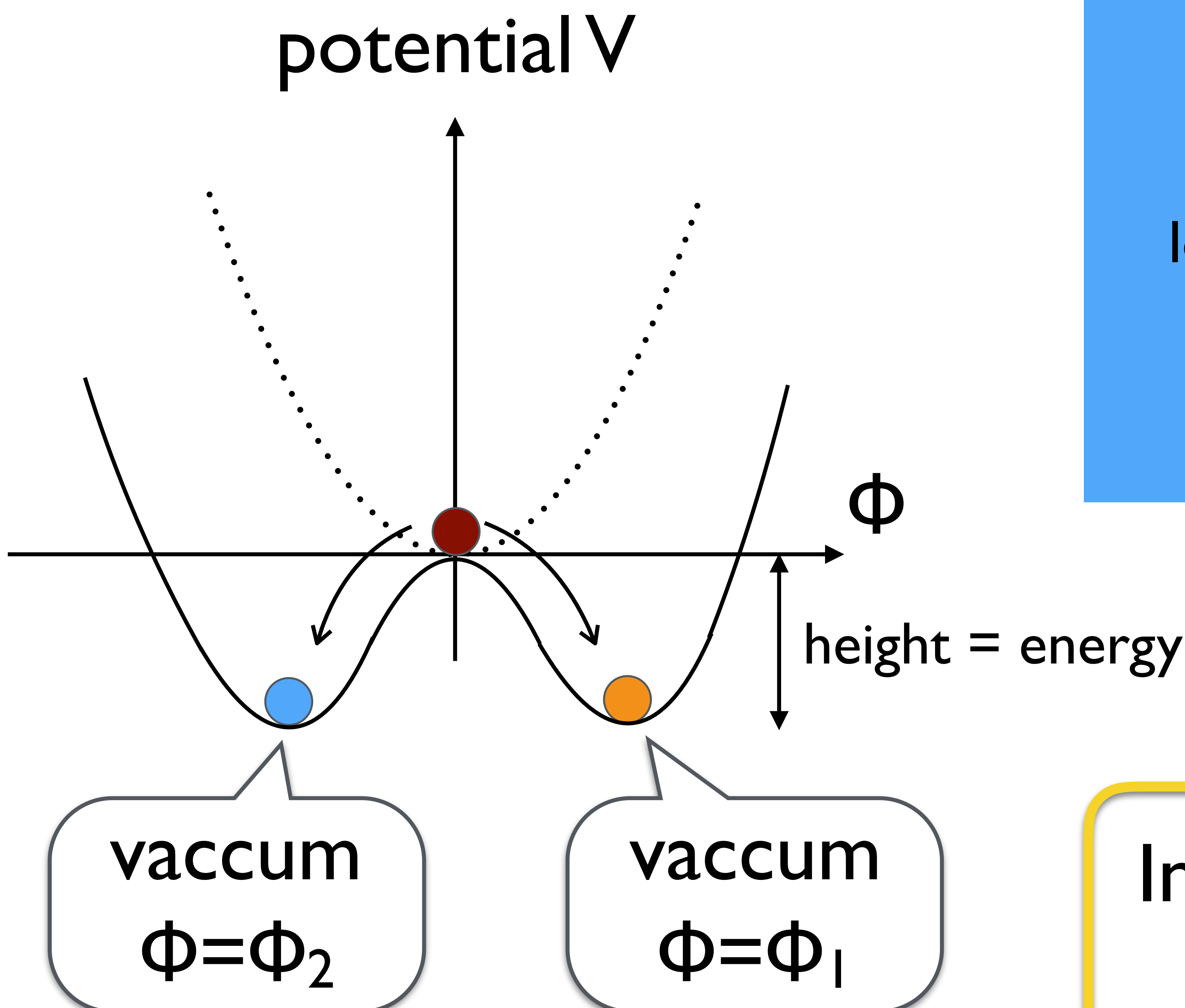


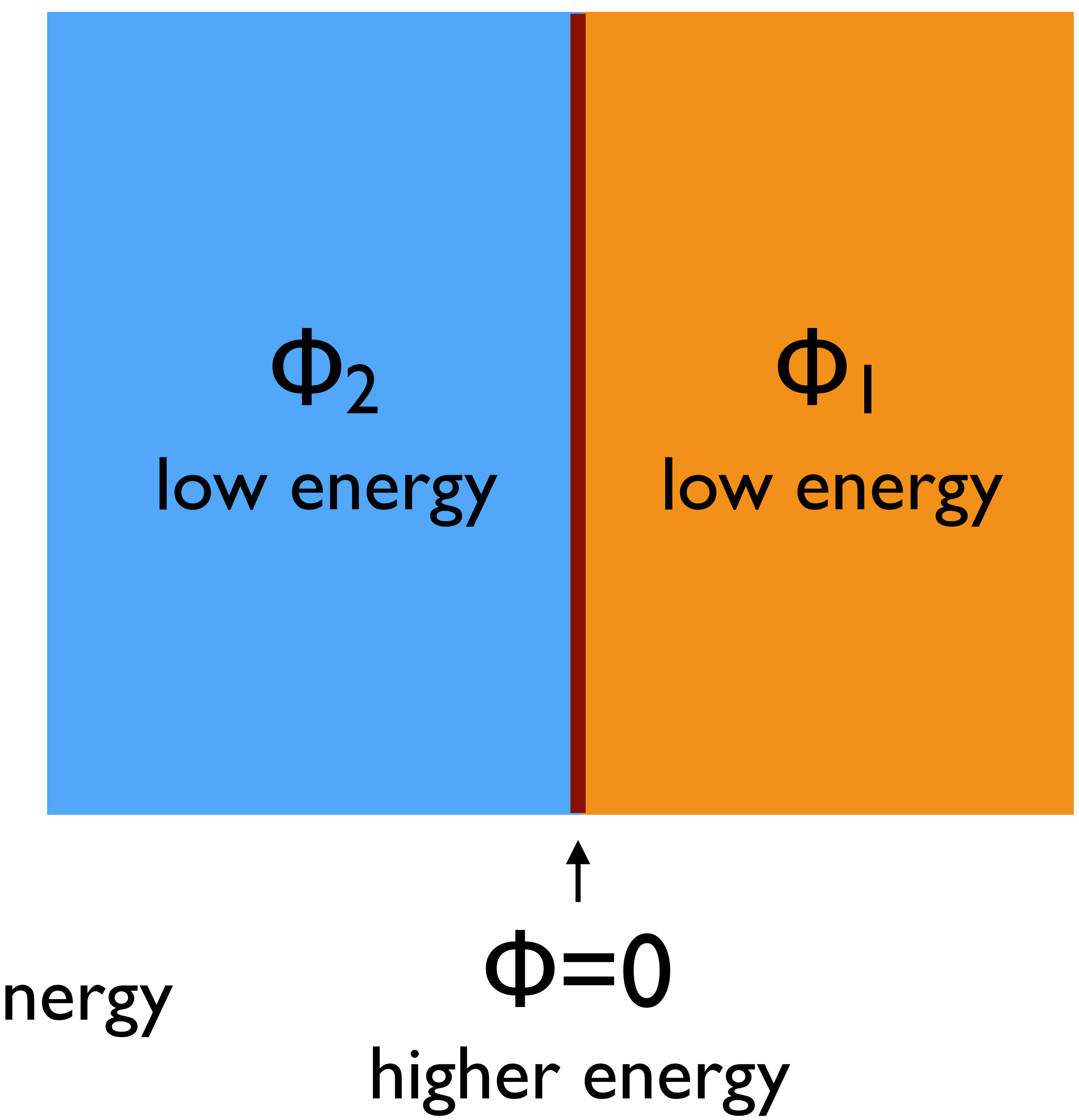
Phase transition in the Universe

Phase transition is described
by the change of the potential



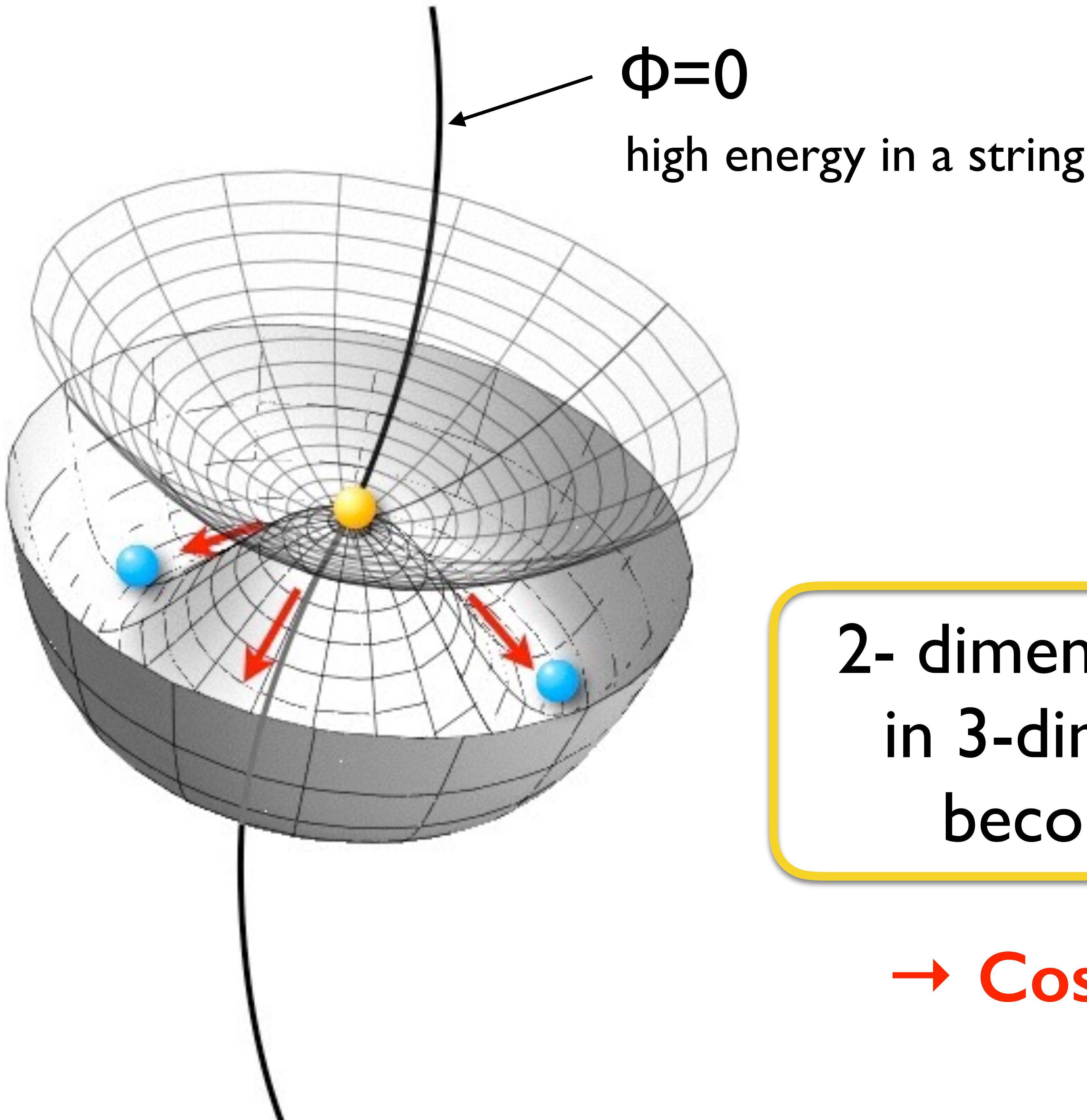
Phase transition in the Universe





In 3-dimension space, it looks like a wall

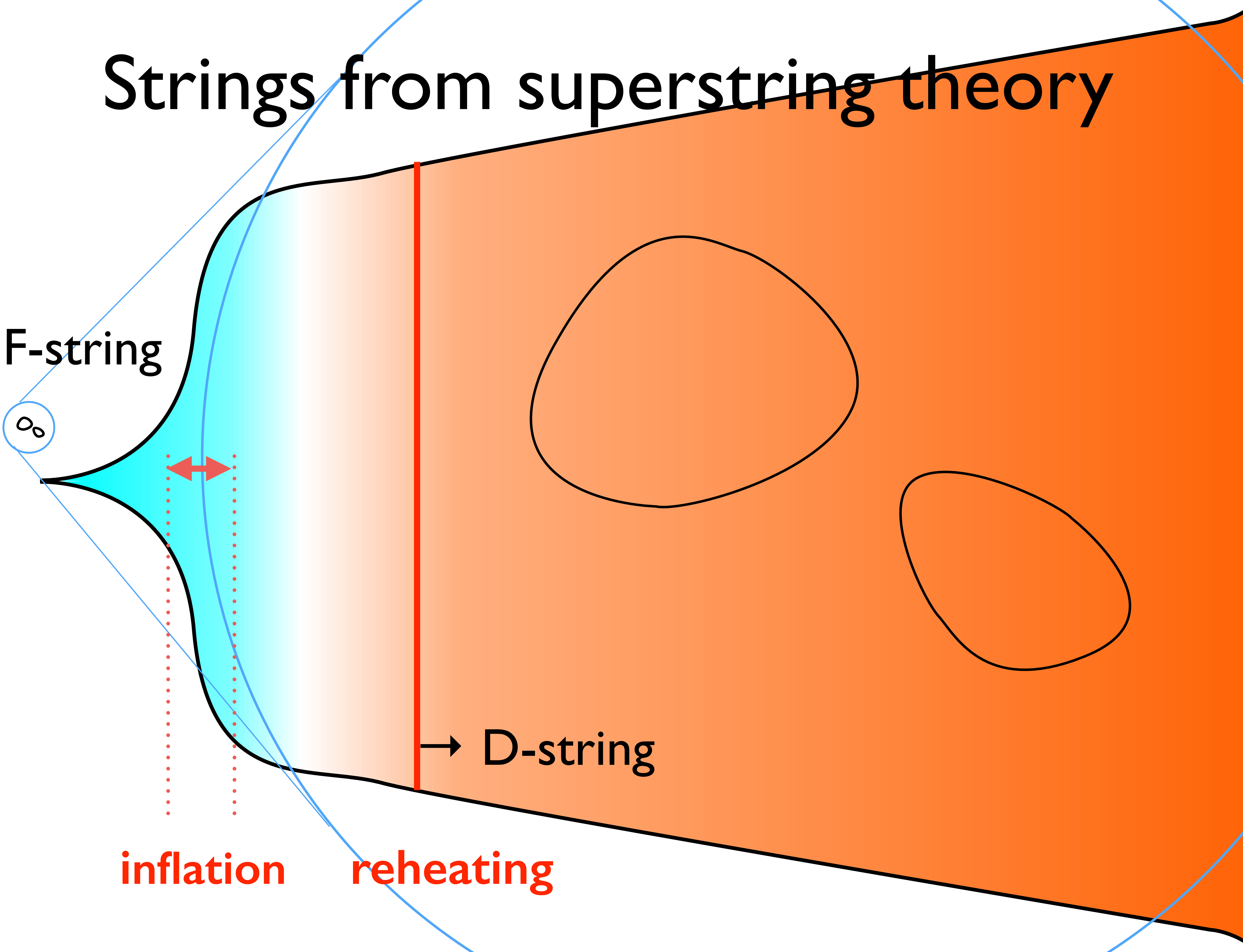
Phase transition in the Universe



2- dimensional potential in 3-dimension space becomes a string

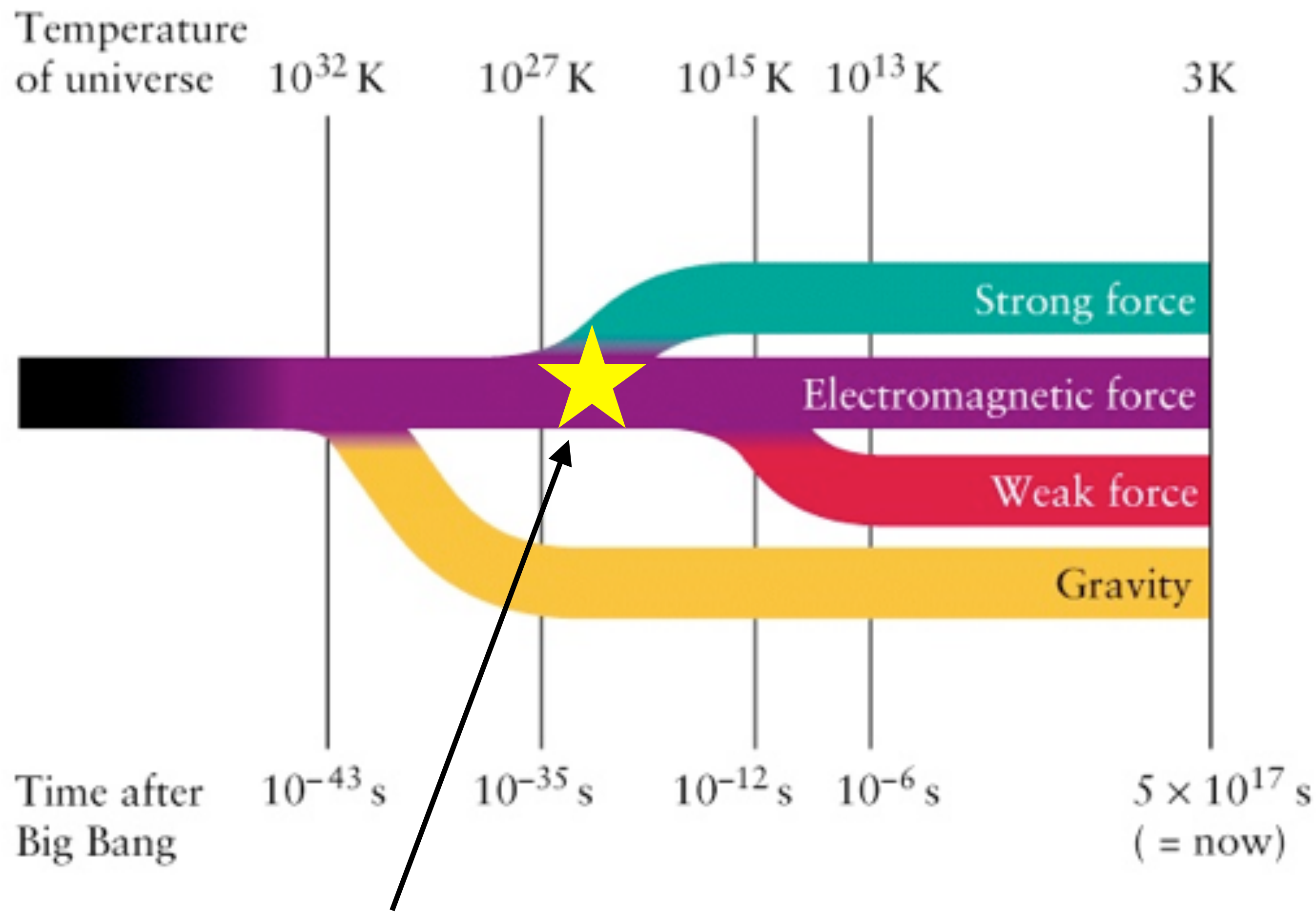
→ Cosmic string

Strings from superstring theory



Important notes

1: Phase transition



"In all acceptable spontaneous symmetry breaking schemes, cosmic string formation is unavoidable"

Jeannerot et al., PRD 68 103514 (2003)

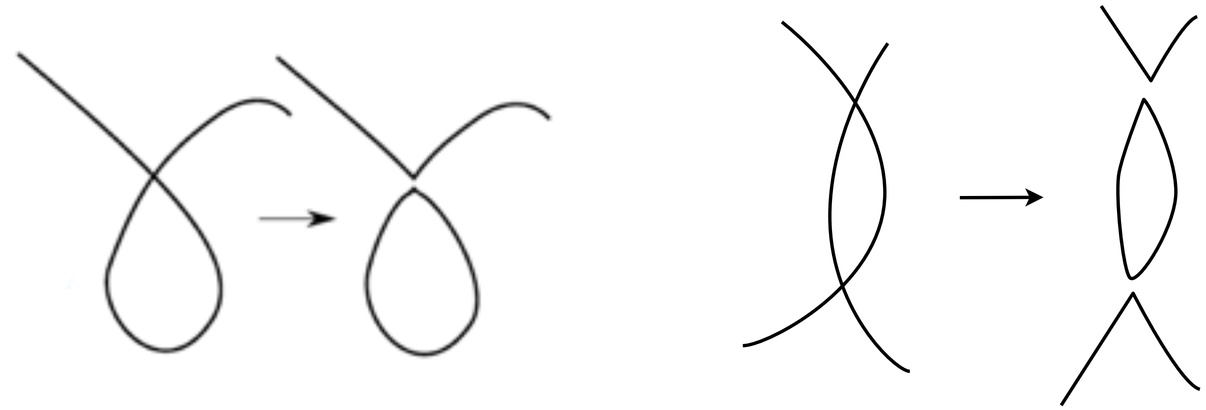
2: Cosmic superstrings

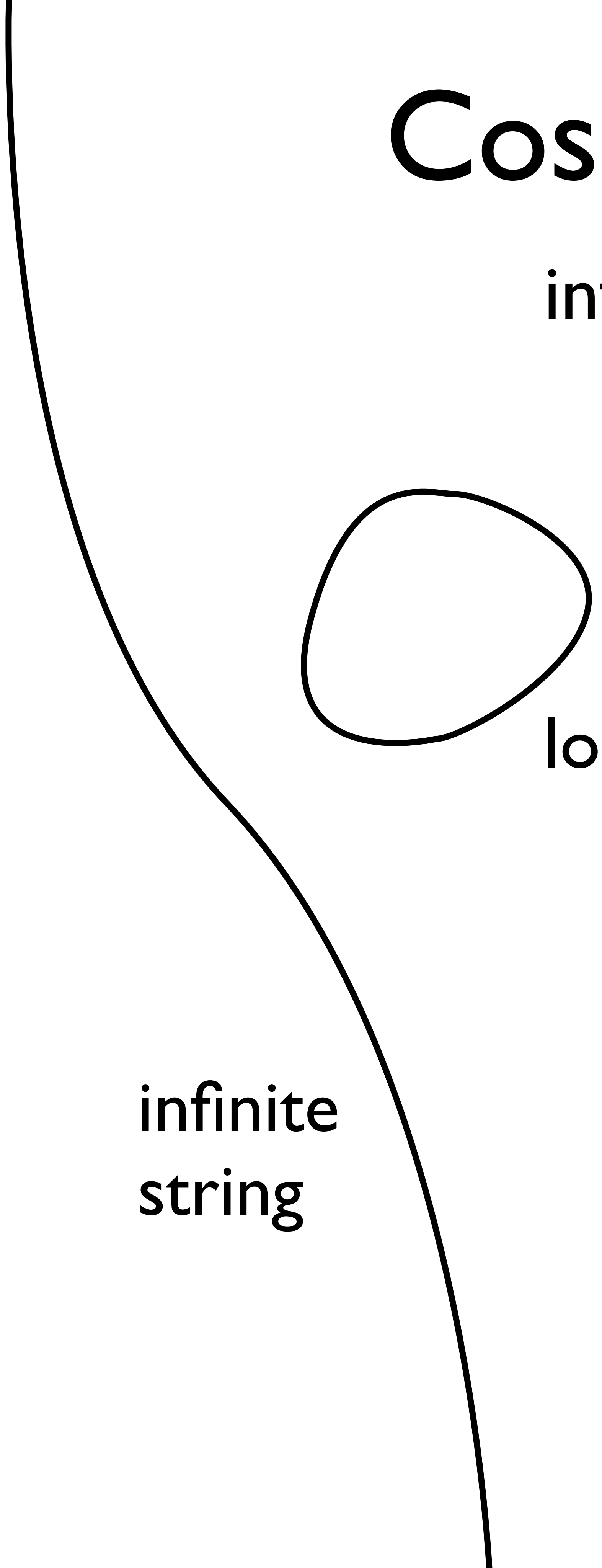
"The remarkable fact is that, although many possibilities remain, every one of them predicts the formation of topological or embedded cosmic strings at the end of inflation. So it seems that cosmic strings are almost unavoidable."

Kibble, Lecture at COSLAB 2004, arXiv:astro-ph/0410073

Cosmic string network

infinite string becomes a loop by reconnection



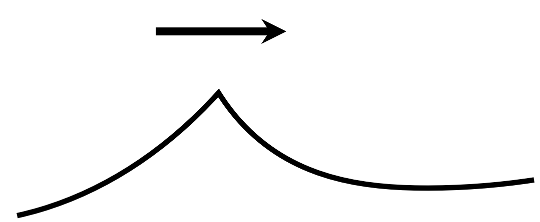


infinite string

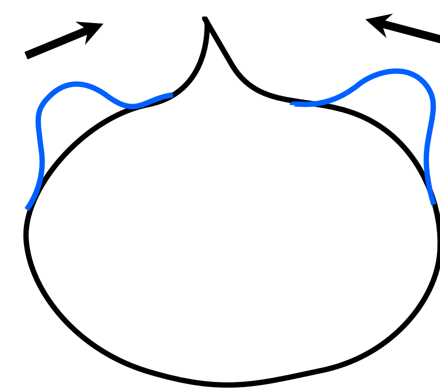
loop

strings emit gravitational waves especially from singular structures

kink

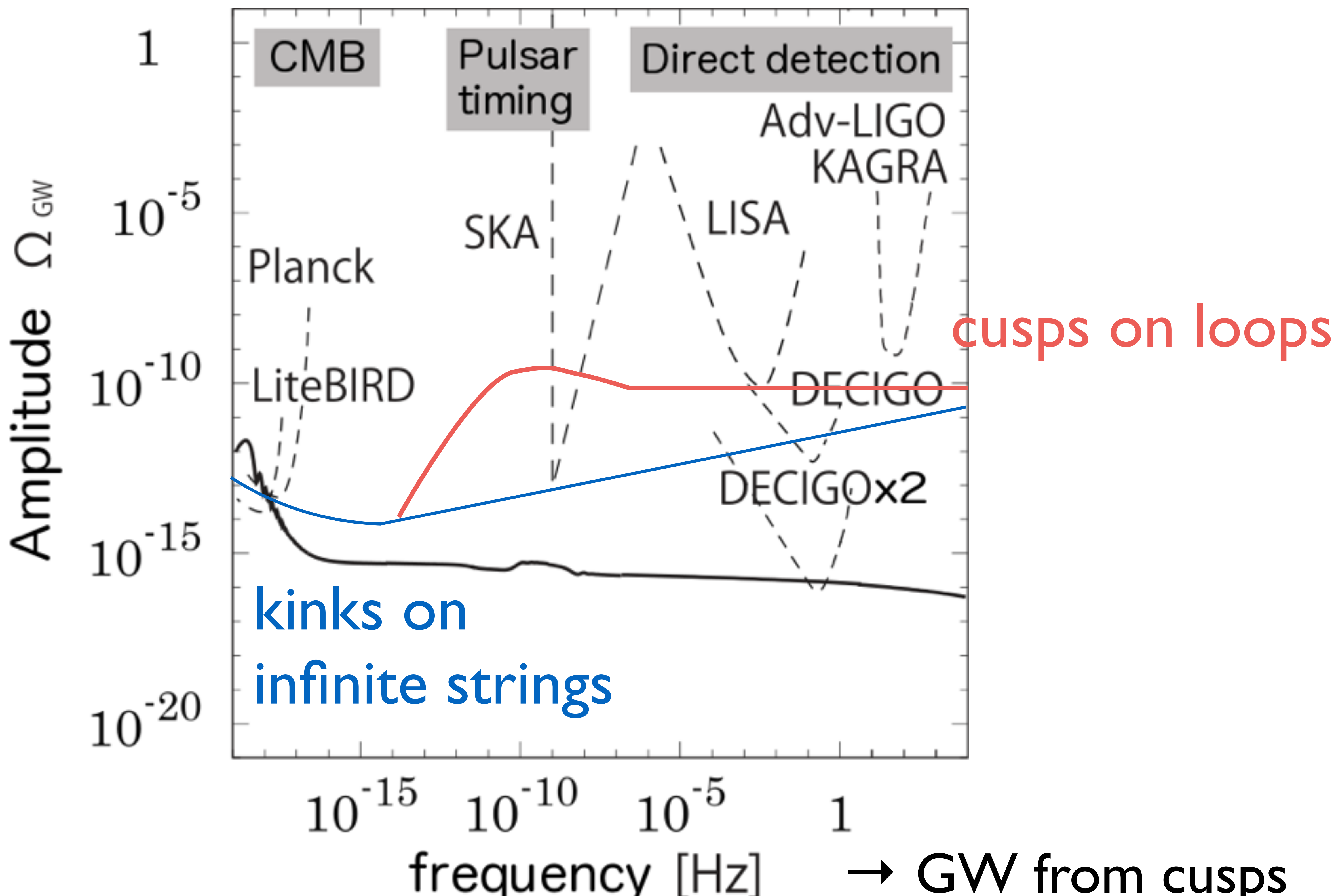


cusp



loops lose energy and shrink by emitting GWs and eventually evaporate

GWs from cosmic strings



→ GW from cusps are dominant

What determines the GW amplitude?

3 main parameters to characterize cosmic string

- **$G\mu$** : tension = line density

  Generation mechanism

- α : initial loop size $L \sim \alpha H^{-1}$

  Network evolution

- **p** : reconnection probability

  Phase transition origin: $p=1$

  Cosmic superstring: $p<<1$

What determines the GW amplitude?

3 main parameters to characterize cosmic string

- **$G\mu$** : tension = line density
  → amplitude of single GW, lifetime of loops
- **α** : initial loop size $L \sim \alpha H^{-1}$
  → number density & lifetime of loops
- **p** : reconnection probability
  → number density of loops

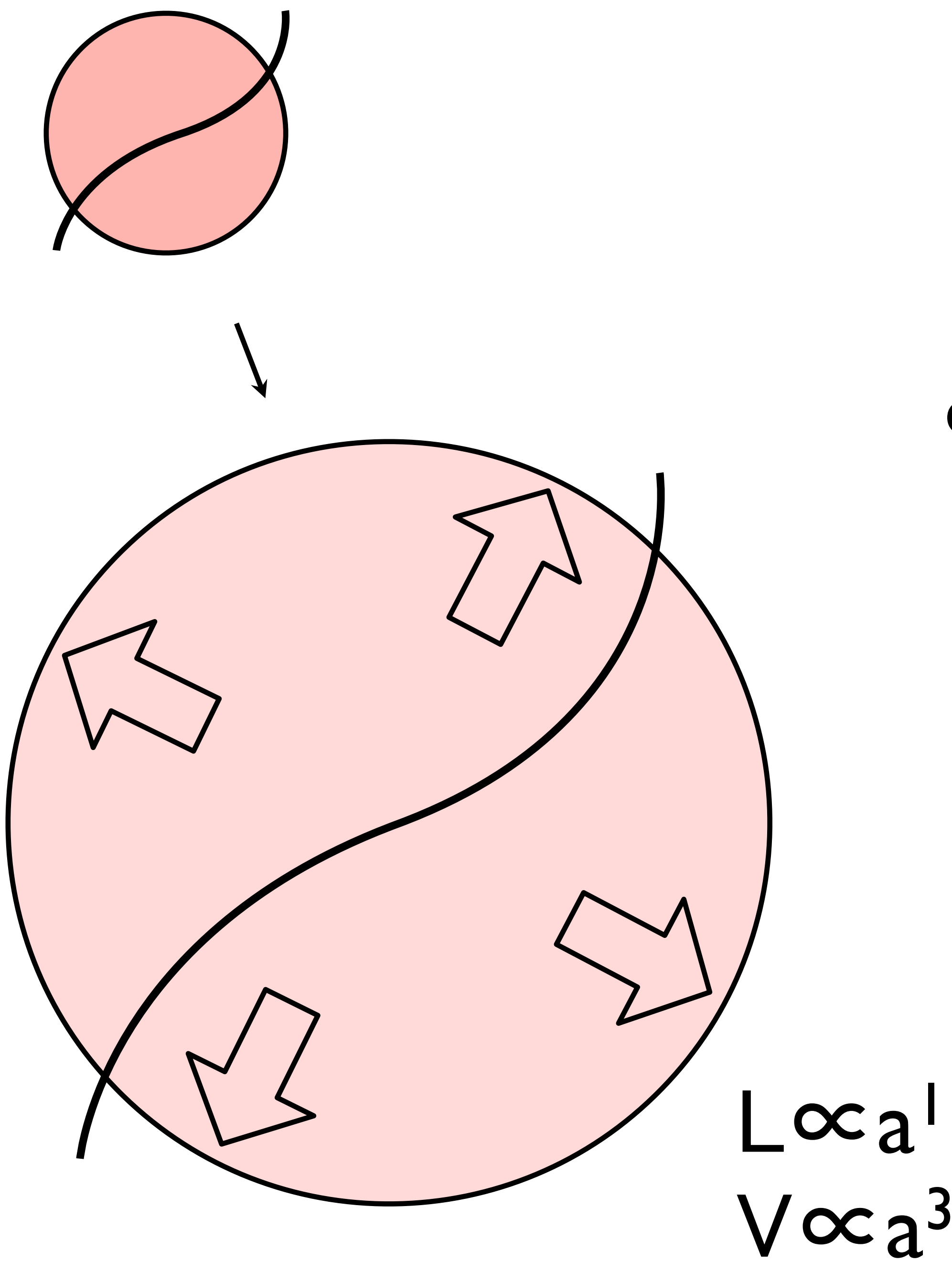
Evolution of cosmic string network

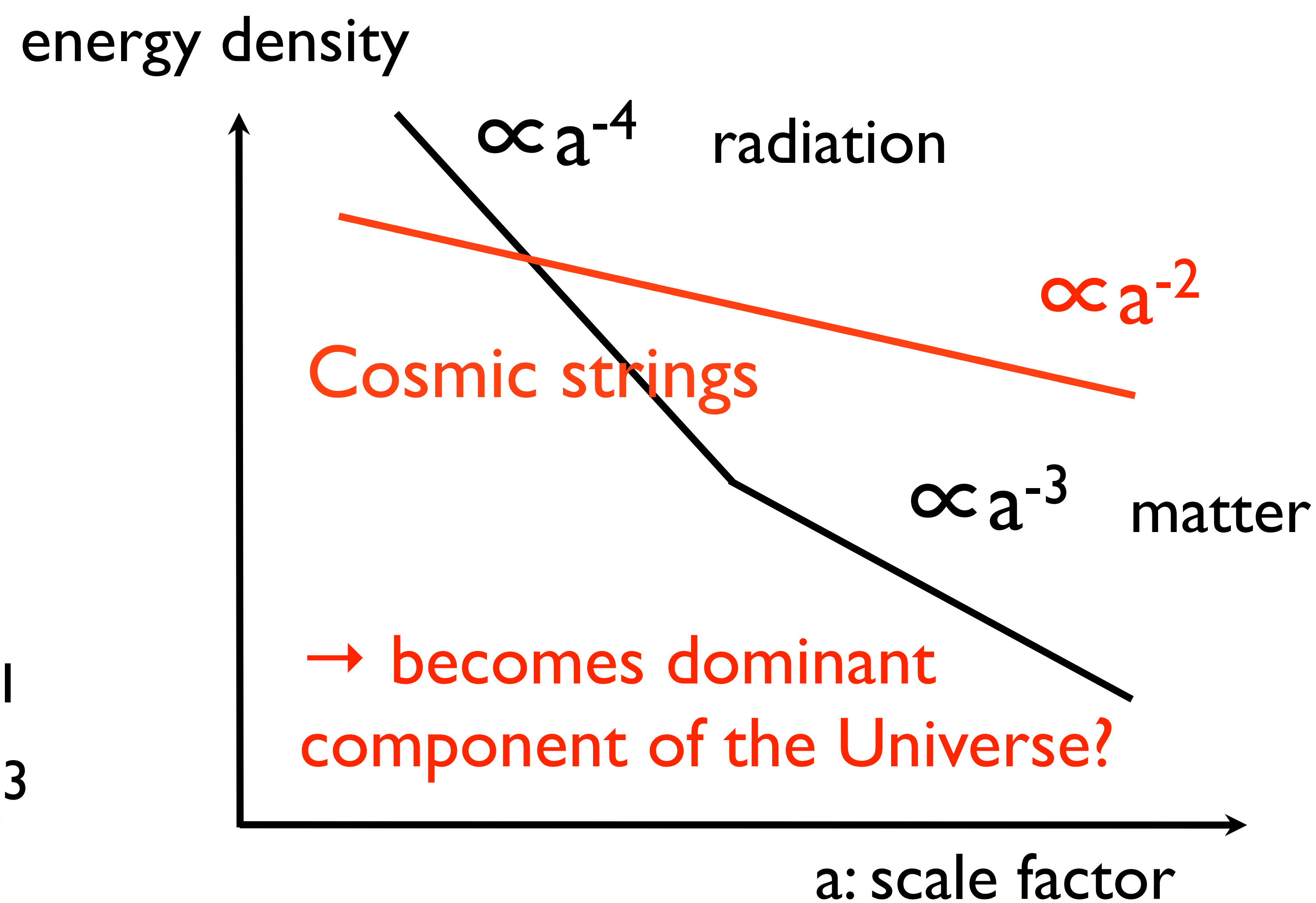
The energy density of cosmic strings

$\sim$ (line density × length)/volume $\propto a^{-2}$ X

$\propto a^{1}$ $\quad$ $\propto a^{3}$

a: scale factor of the Universe





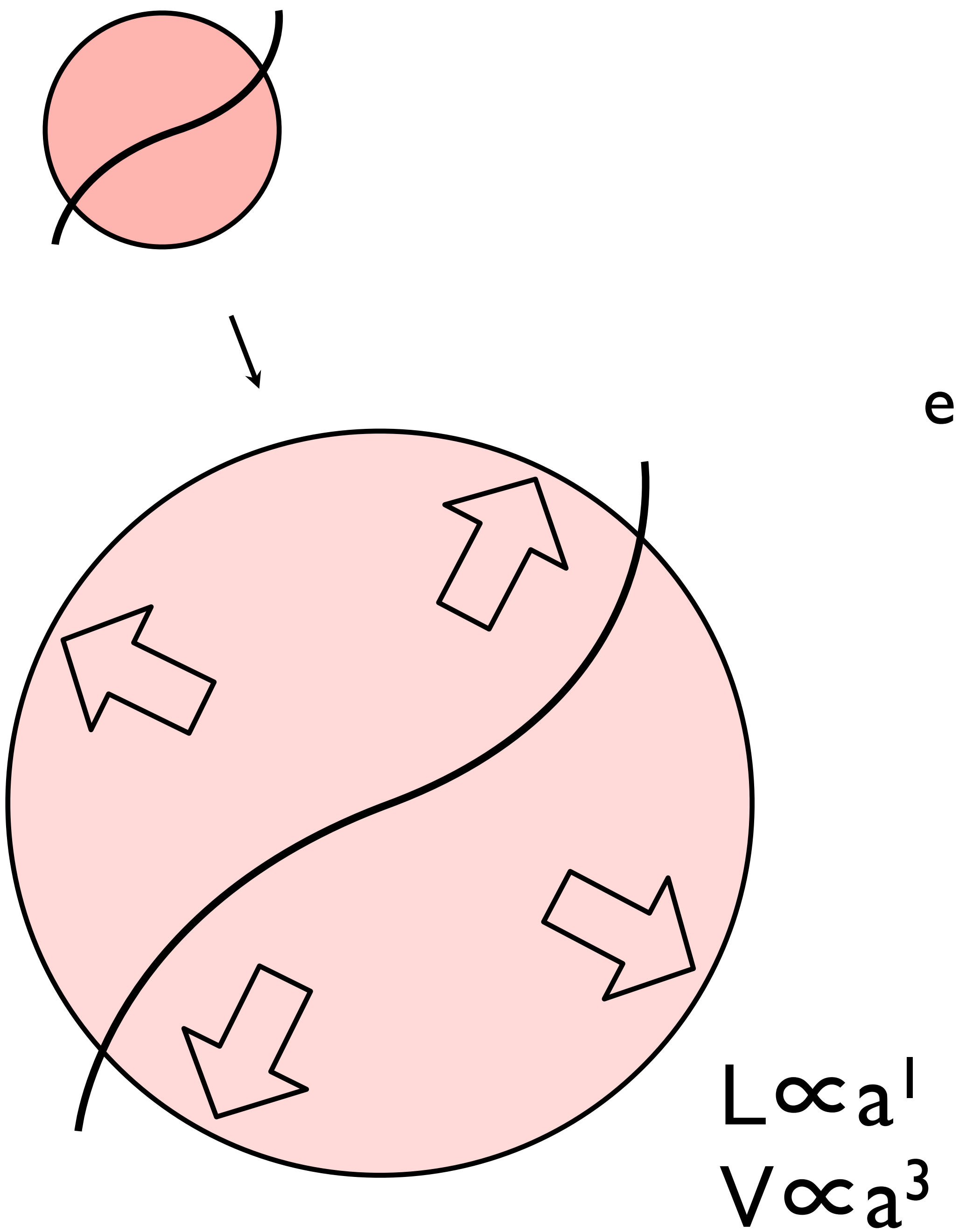
Evolution of cosmic string network

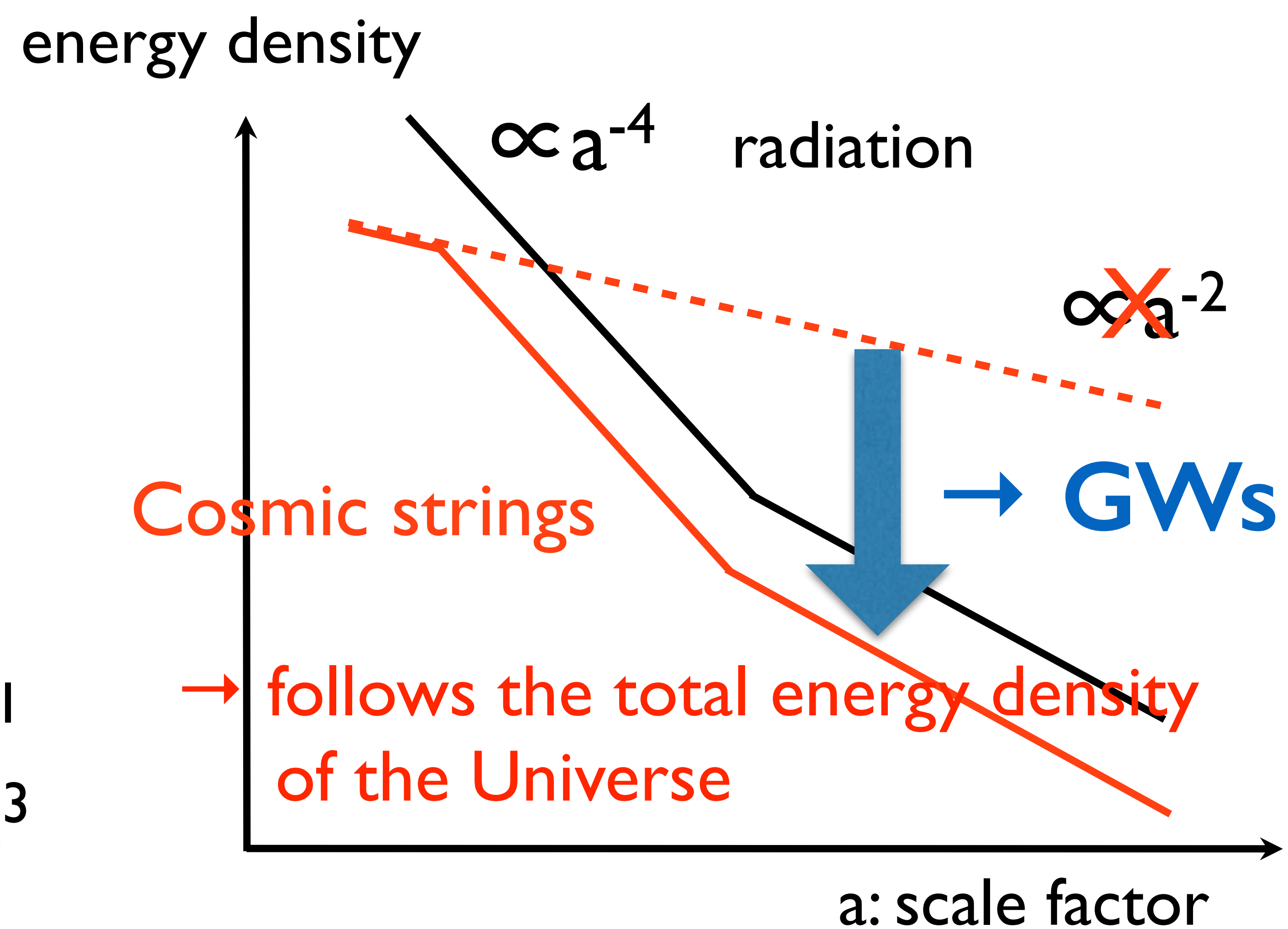
The energy density of cosmic strings

$\sim$ (line density × length)/volume $\propto a^{-2}$ X

length $\propto a^{1}$, volume $\propto a^{3}$

a: scale factor of the Universe



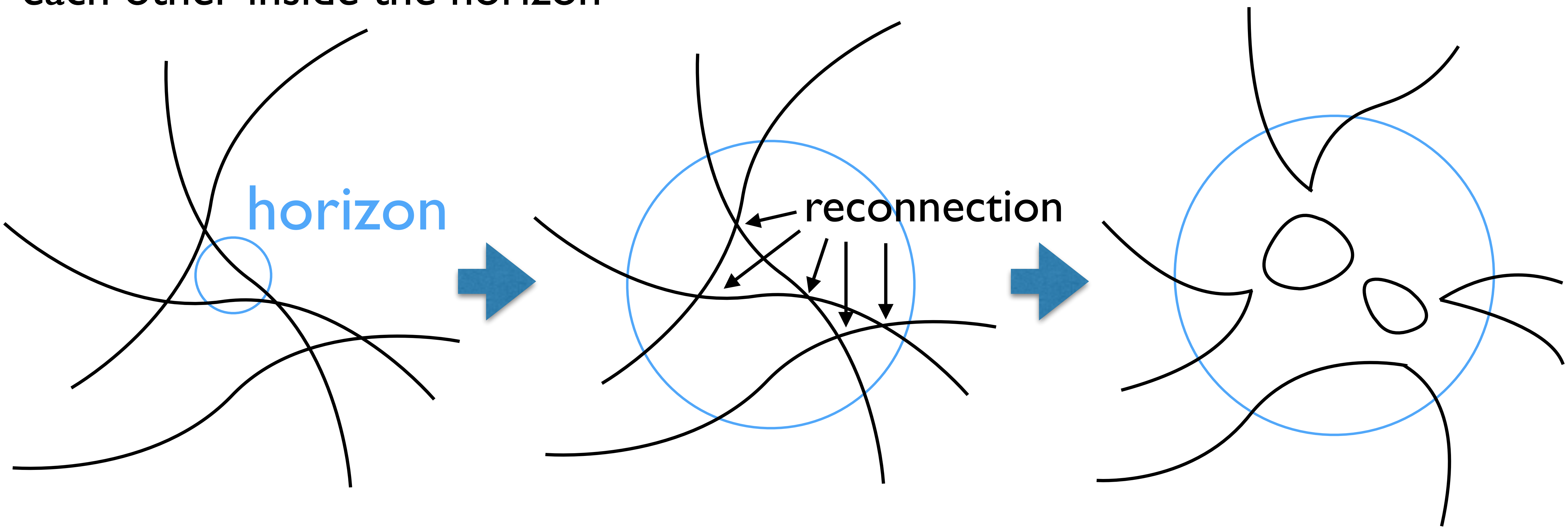


Evolution of cosmic string network

<u>Scaling law</u> The Universe always has O(1-10) strings per horizon

String network keeps producing loops

Strings can communicate
each other inside the horizon



→ Loops disappear after
continuous GW emission

Evolution of loops

depends on Gμ and α

Initial loop length $= \alpha t_i$

t_i : time when the loop formed

GW power $P = \Gamma G \mu^2$ Γ: numerical constant ～50-100

From the energy conservation law

(energy of loop at time t =μl)

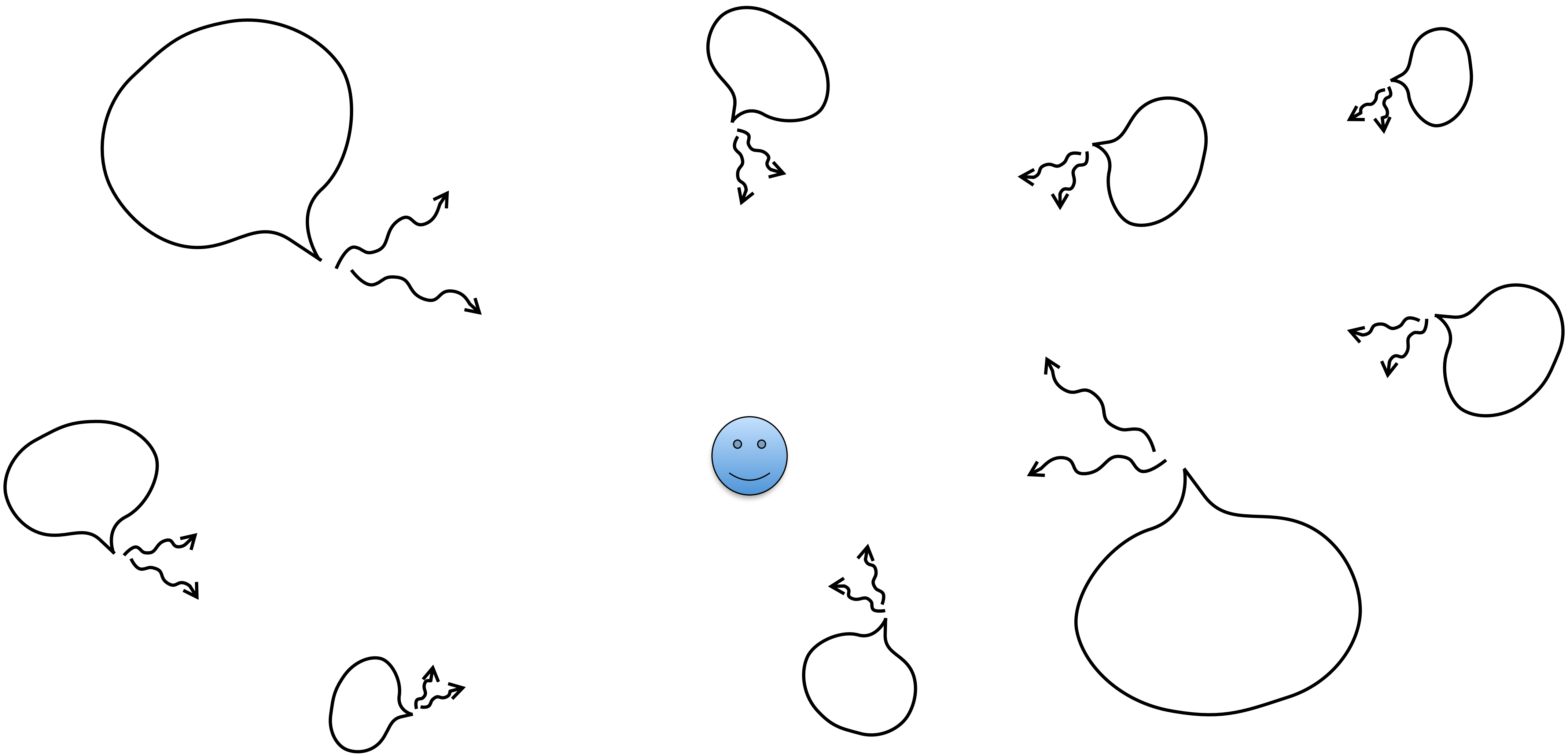
= (initial energy of the loop =μαt_i) − (enegry released to GWs =PΔt)

Loop length at time t $l(t, t_i) = \alpha t_i - \Gamma G \mu (t - t_i)$

Lifetime of the loop $= \dfrac{\text{(initial loop energy)}}{\text{(energy release rate per time)}}$

$$= \frac{\mu \alpha t_i}{\Gamma G \mu^2} = \frac{\alpha t_i}{\Gamma G \mu}$$

Gravitational waves from cosmic string loops



Gravitational waves coming from different directions
overlap each other and form gravitational wave background

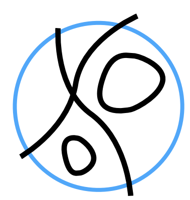
Search for bursts & stochastic background are both important

Wide range GW background spectrum

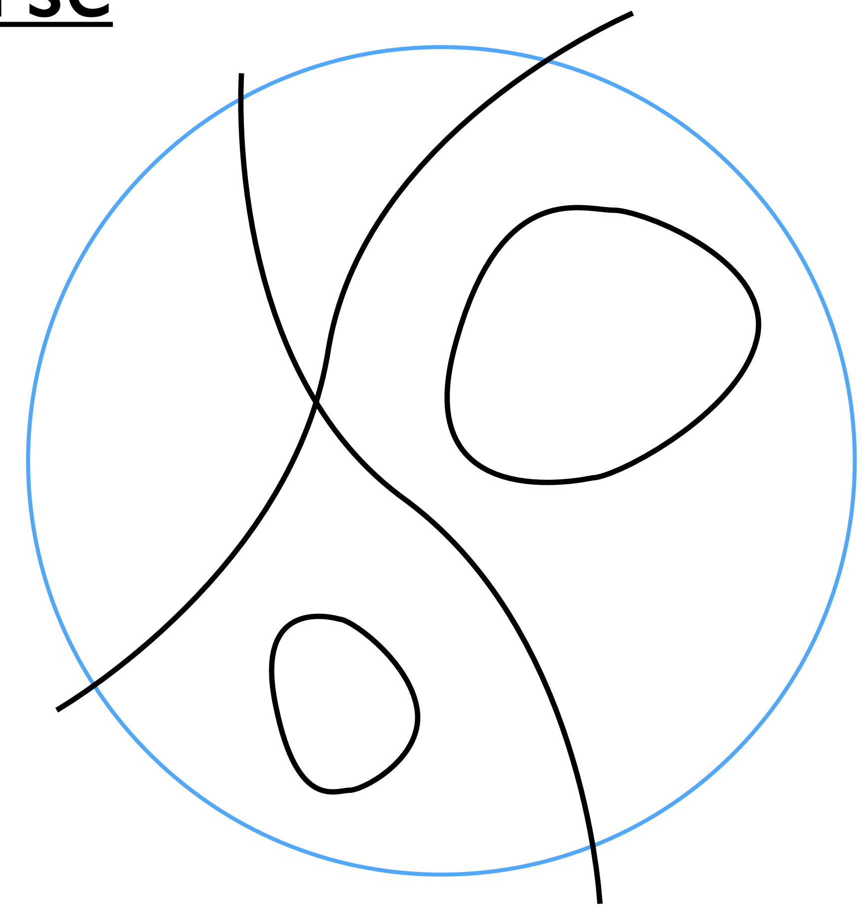
Scaling law The Universe always has O(1-10) strings per horizon

Early Universe

horizon



Late Universe



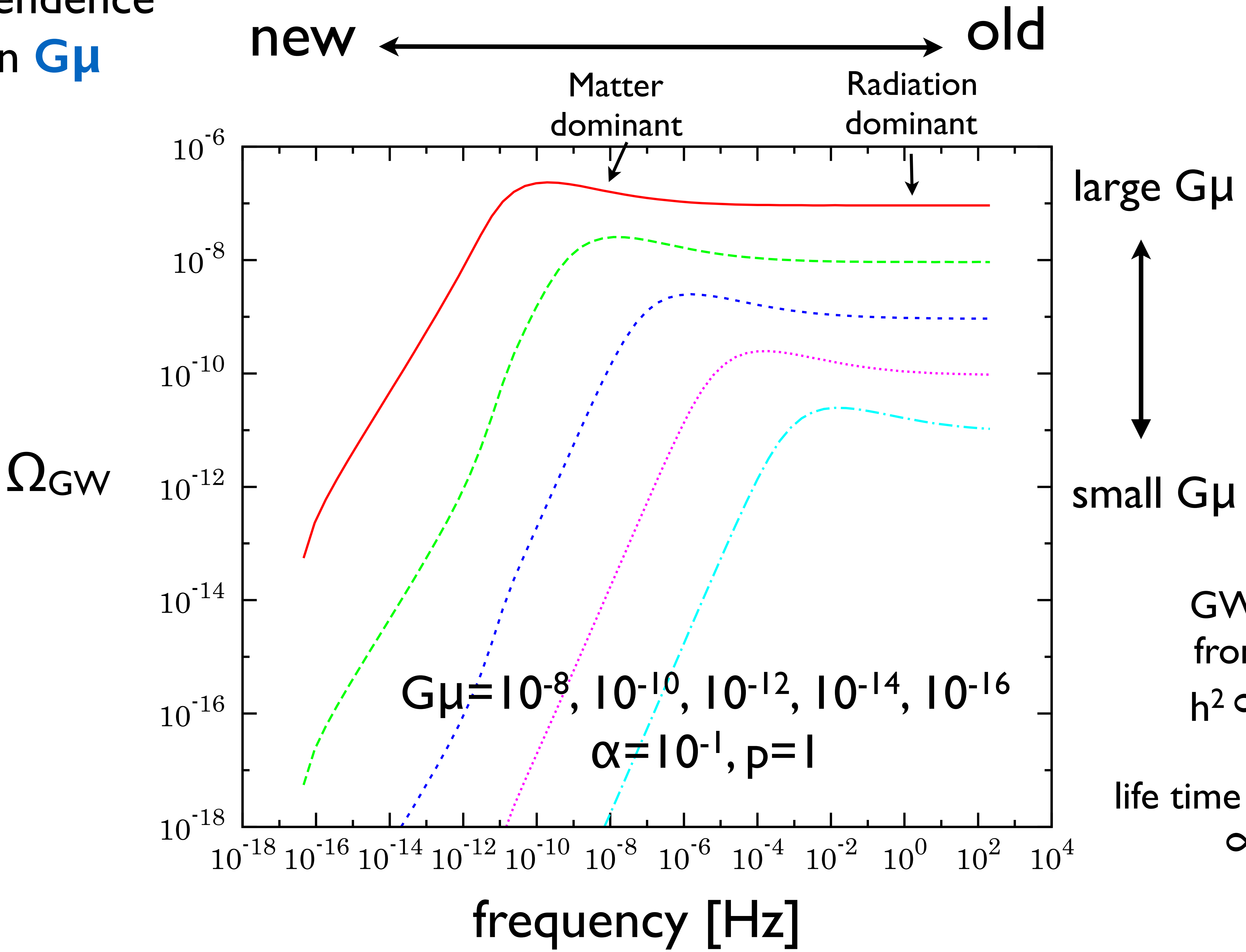
GW frequency ~ 1 / (loop size)

→ high frequency GWs

→ low frequency GWs

Spectrum of the GWB

dependence on **Gμ**



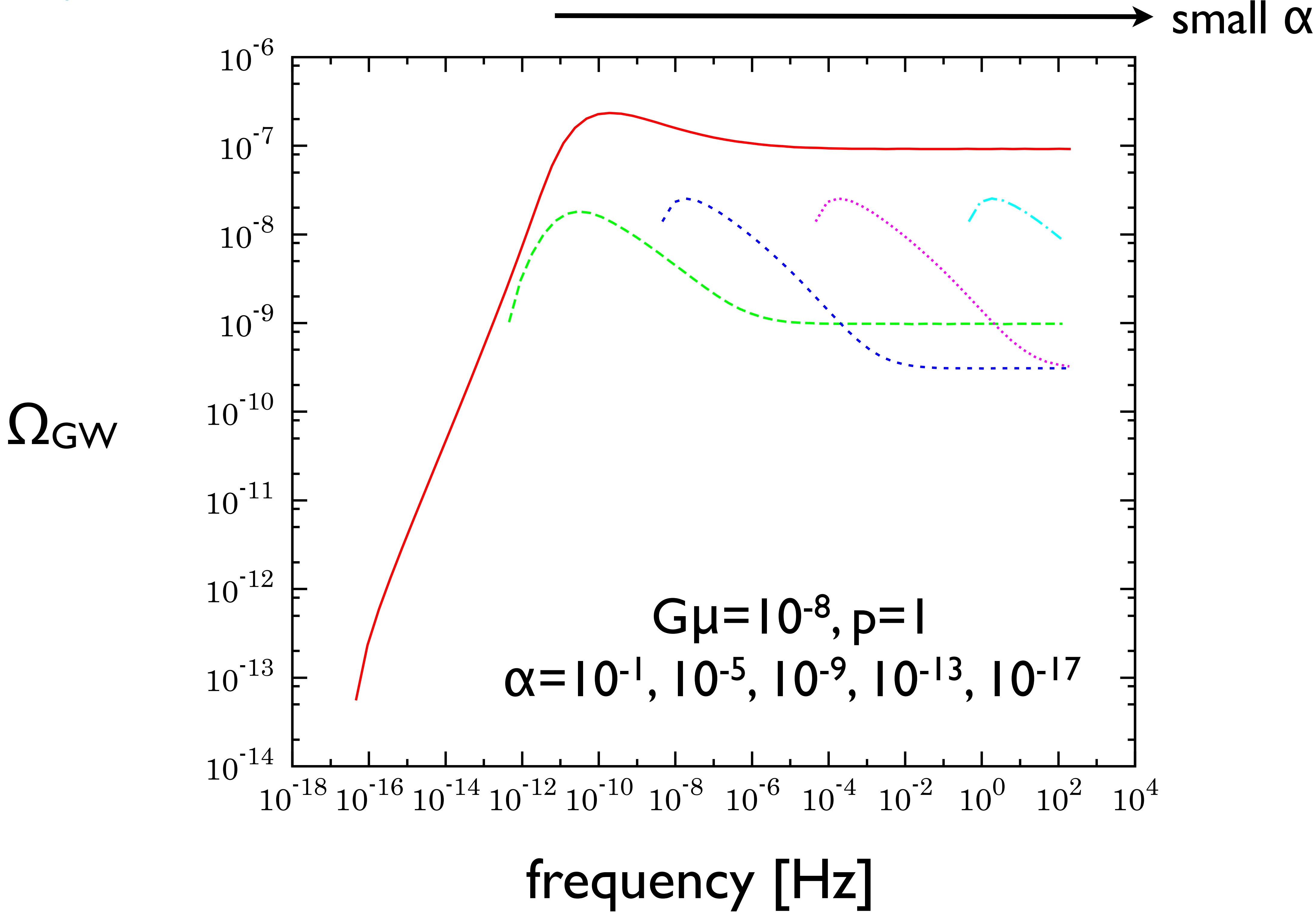
GW power from cusps $h^2 \propto (G\mu)^2$

life time of loops $\propto (G\mu)^{-1}$

Spectrum of the GWB

dependence on **α**

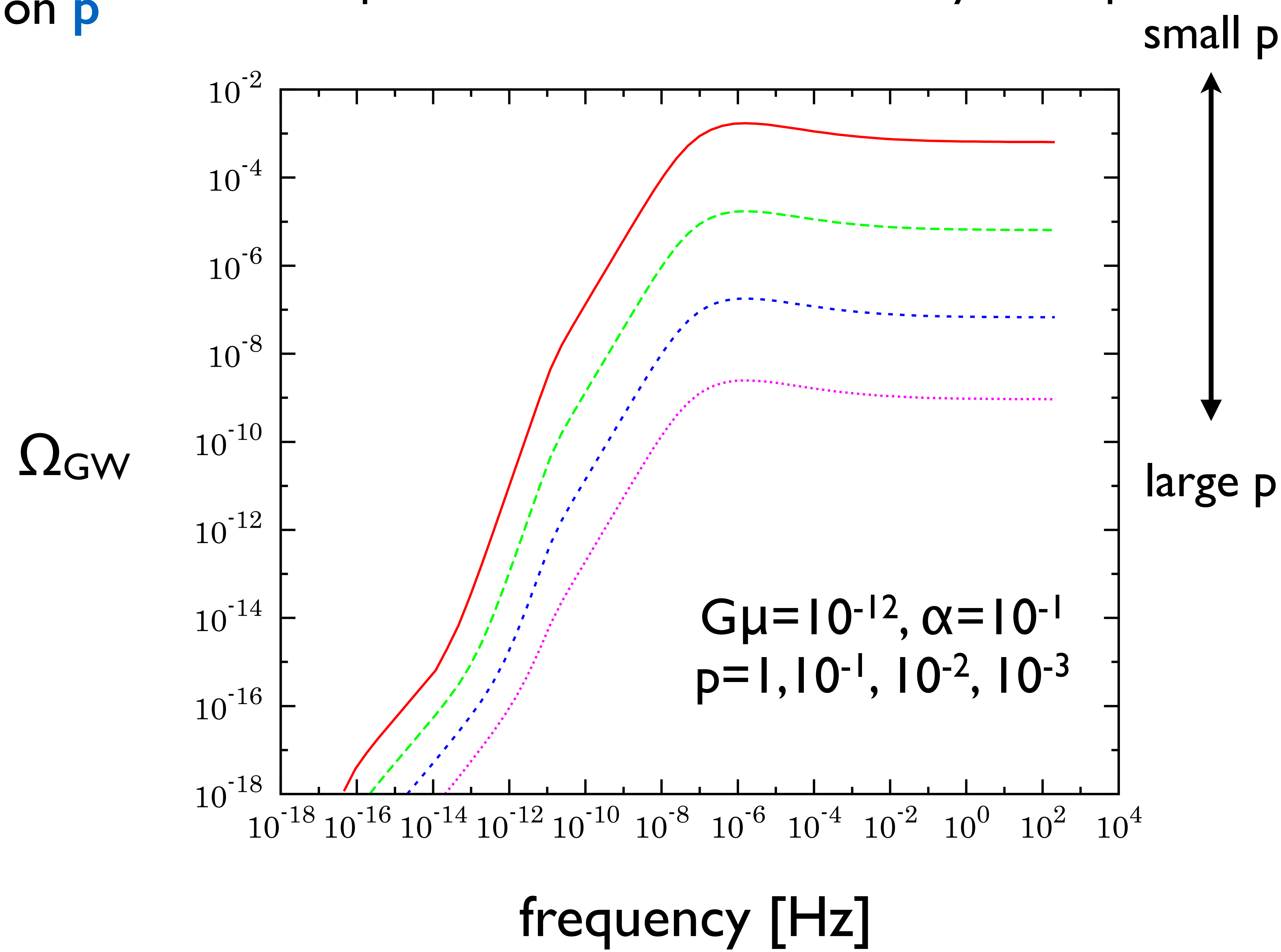
loop size directly corresponds to the frequency of the GW



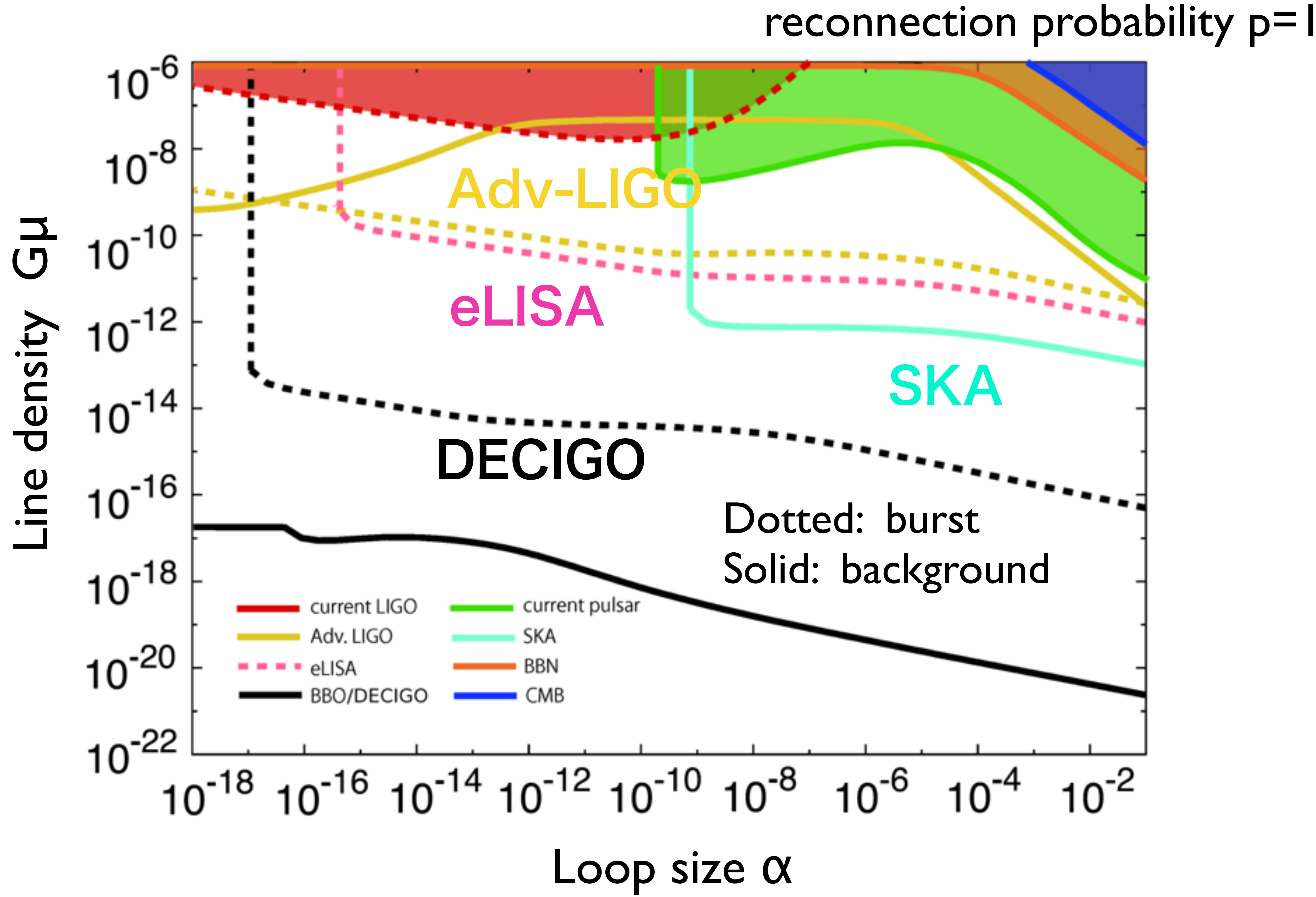
Spectrum of the GWB

dependence on p

small p increases the number density of loops



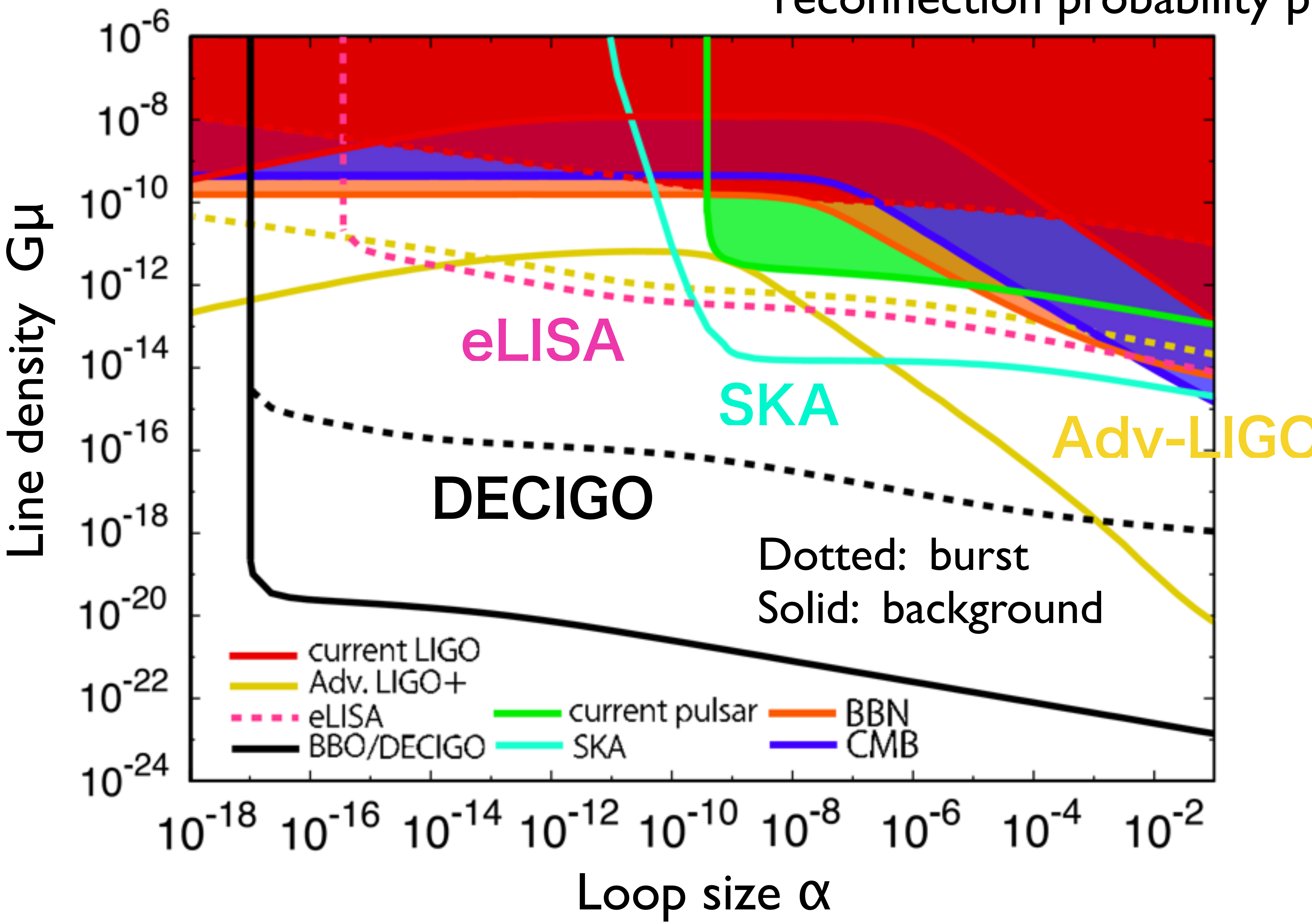
Accessible cosmic string parameter space



Kuroyanagi et al. PRD 87, 023522 (2013)

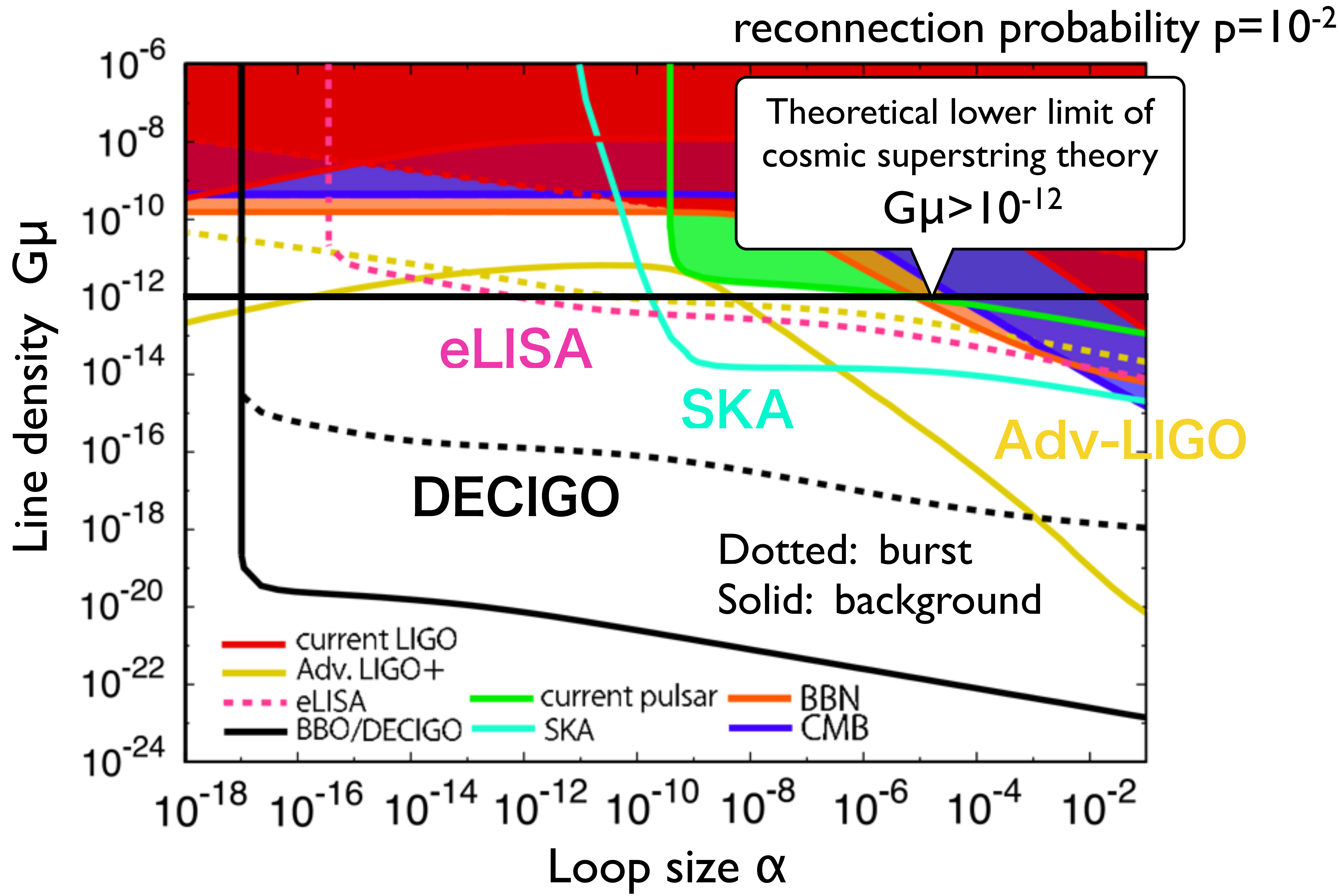
Accessible cosmic string parameter space

reconnection probability $p=10^{-2}$



Kuroyanagi et al. PRD 87, 023522 (2013)

Accessible cosmic string parameter space



Kuroyanagi et al. PRD 87, 023522 (2013)

Summary

GWs can become a powerful probe of the very early Universe

- GWs from inflation are promising

  (but the amplitude may be small)

  → It may provide information not only on inflation but also on the thermal history after inflation (eg. reheating)

- GWs from cosmic strings has relatively large

amplitude and testable by near-future experiments

→ It will be useful constraint for the model building of the early Universe


New window of observation
has just opened
Many things to explore in future!